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Reviewers

The individuals below have given their time and expertise to read and review manuscripts submitted for this edition of the *Banneker Banner*. We are very grateful for their help.

***Banneker Banner* Submission Guidelines**

The Banneker Banner is the official journal of the Maryland Council of Teachers of Mathematics. The journal is named after Benjamin Banneker, a Maryland native and perhaps the first documented African-American mathematician. The Banner is published once or twice each year and contains a wide range of articles on issues in mathematics education at all levels. Articles published in the journal must be submitted to the editor and undergo a peer review process.

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. All submissions should be relevant, interesting, and useful to teachers of mathematics in Maryland. More information about the submission process can be found here.

Developing Understanding of Fraction Equivalence

Mackenzie Peperak, Washington County Public Schools
Jathan Austin, Salisbury University

Abstract: *In this article, we discuss how rising fourth graders' understandings of fraction equivalence developed over the course of a ten-week summer program. We found that all students steadily improved in their abilities to give conceptual explanations during the study, yet most struggled with equivalent fractions that could not be formed from a given fraction by halving or doubling the numerator and denominator. We found that the use of discrete manipulatives helped students with these difficulties.*

Introduction

Many students develop procedural fluency with equivalent fractions; however, difficulties with developing conceptual understandings of fraction concepts are well-documented (Geller et al., Mack, 1990; 2017; Simona et al., 2018). Students learn procedures for working with equivalent fractions. They may be able to generate sets of equivalent fractions, list fractions that are equivalent to a given fraction, or verify that two fractions are equivalent. Understanding why fraction-equivalence procedures work and developing deep understandings of what it means for two fractions to be equivalent can be difficult.

Children may find certain strategies helpful for navigating equivalent fractions. The "splitting" (or "partitioning") strategy involves constructing an equivalent fraction by splitting each of the parts of a given fraction into the same number of equal-size pieces (Izsak et al., 2018). For example, ¾ can be transformed into 6/8 by splitting each fourth in half. In a "chunking" strategy, equal numbers of parts are grouped or "chunked" together to construct an equivalent fraction (Empson, 2001). 5/10 becomes ½ when tenths are "chunked" together.

We recently had an opportunity to help children develop their conceptual understanding of fraction equivalence during a series of lessons that led to the use of splitting and chunking strategies. In the next section, we report our experiences, which took place with a small group of children during a summer research project.

Our Study

Our study took place in the context of a 10-week summer program during which lessons for the following week were designed according to the cycle shown in Figure 1. Four students who had just finished third grade prior to the summer program participated in the fraction-equivalence lessons.

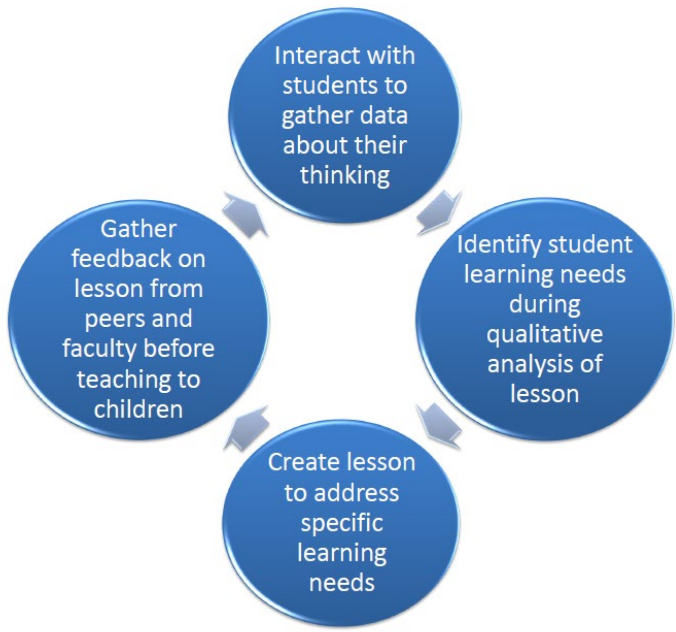

Figure 1. The Instructional Cycle.

Lessons 1 and 2

During our first lesson we wanted to gauge what the children already understood about fractions. We had students try to recognize fractions from a visual representation, then create their own fractions and draw corresponding representations. In order to see how deep our students' understandings of fraction equivalence were, we asked them to list pairs of fractions that they thought were equivalent. Students used paper and pencil and simply wrote out a fraction as well as one they deemed to be equivalent. We did not provide manipulatives for this activity because we wanted to see what understandings students would exhibit independent of the aid that manipulatives offer. We found that the children generated both correct and incorrect examples (e.g., see Figure 2).

Figure 2. Examples of correct and incorrect equivalent fraction pairs given by a student.

This showed us that our students had incomplete understandings of fraction equivalence and how to tell when fractions are equivalent. We knew, in order to develop a conceptual understanding of fraction equivalence, our students must first have a correct understanding of what it means for fractions to be equivalent.

During lesson two, students were given several pairs of fractions. Some were pairs of equivalent fractions, but we also included the incorrect examples students generated during lesson 1 to provide students with opportunities to address their prior misconceptions. Students were asked to determine whether each given pair of fractions was equivalent using fraction bars. We found that the fraction bars helped three of our four students recognize which pairs represented equivalent fractions. The fourth student demonstrated difficulty comparing the fraction bars to determine equivalence. In later lessons, this student showed growth in utilizing manipulatives to determine equivalence. In future lessons, we encouraged students to continue to use fraction bars to compare fractions to determine whether they were equivalent.

Lessons 3 and 4

For the next two lessons, we began using fraction strips to help students further develop understanding of fraction equivalence. While fraction strips are similar to fraction bars, we wanted to provide students with a tactile resource to supplement their written representations of fractions. Students cut out and labeled the fraction strips themselves, then used them to find pairs of equivalent fractions. For example, students were able to use a “lining up” strategy to see that two pieces representing one-sixth were the same size as one piece representing one-third.

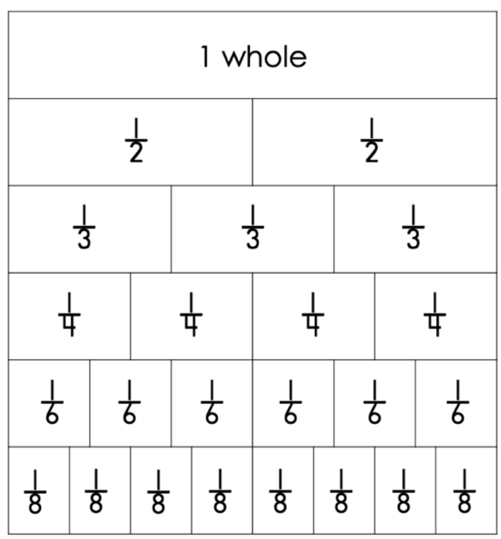

Figure 3. Fraction strips used by students.

This activity prompted an inquiry-based approach to finding equivalent fractions. Students were able to ask questions about what fraction strips were equivalent and find the answers themselves. This promoted curiosity and ownership of their mathematical learning. For example, if a student was curious about how many fractions they could make that were equivalent to one-half, they had the resources to explore those relationships and discover the answer independently.

For lesson four, we wanted students to develop a deeper understanding of why fractions are equivalent, and to realize that multiple fractions can be seen within a single visual representation. Figure 4, for example, shows how one student was able to recognize 12/16 as an equivalent fraction from the given representation of 6/8. The student identified 2/8 as a possible representation by finding the value of the unshaded area of the rectangle.

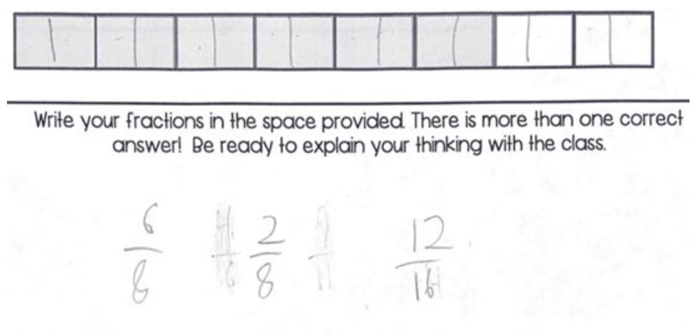

Figure 4. Student work showing multiple equivalent fractions from one visual representation.

We had noticed our students tended to rely on unit fractions, fractions that have a numerator of one. Our students could figure out that 1/3 was equivalent to 2/6 but could not figure out that 3/9 was also equivalent to 2/6. In order to encourage students to explore equivalent fractions with differing numerators we began to work with fractions that could not be simplified into unit fractions. When we took away the possibility of finding unit fractions, the students naturally began to use “splitting” and “chunking” strategies. One student showed both splitting (by drawing vertical line segments) and chunking (by grouping the eighths into pairs) in the same diagram, as shown in figure 5. The student realized that by splitting each equal-size box in half, the numerator and denominator of their original fraction, 6/8, doubled and the new fraction became 12/16. When prompted to find a second strategy for finding an equivalent fraction for 6/8 the student decided to combine the boxes into pairs. The student was then able to demonstrate that 3/4 is equivalent to 6/8. Discovering these strategies was an important step for our students as it allowed them to independently discover a reliable way to find equivalent fractions with differing numerators using a diagram.

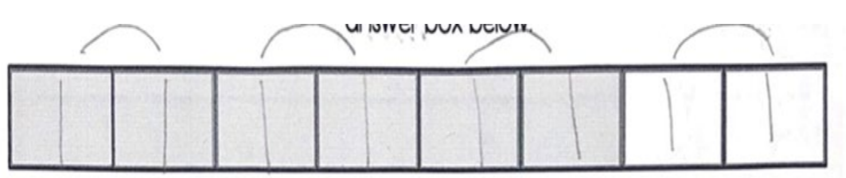

Figure 5. Student work showing beginning understanding of splitting and chunking ideas.

Lessons 5, 6 and 7

During the final three lessons, students further developed their abilities to reason about patterns and gave explanations with mathematical language. Students were much more likely to use the terms “splitting” and “chunking” when explaining their strategies when they could split or chunk physical objects into groups. In lesson five we used buttons to help with this; students created fractions using different button attributes, such as grouping buttons together according to color, number of holes, or size.

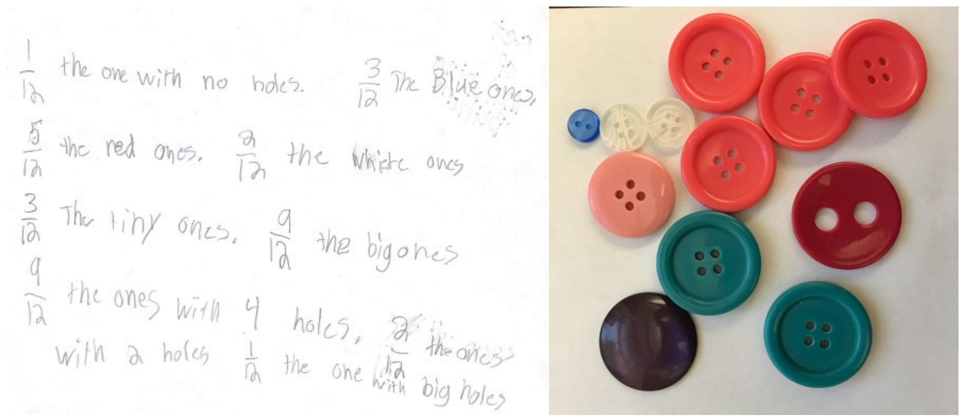

Figure 6. A student’s generated fraction examples using buttons.

Figure 6 shows one student’s work using the chunking strategy. The student had 12 buttons and chunked them into different groups based on as many characteristics as they could find. For example, when looking at how many tiny buttons they had, the student created numerators of three for tiny buttons and nine for larger buttons.

In lesson six, we used similar activities using snap cubes. We noticed that even though students used splitting and chunking strategies to build pairs of equivalent fractions, they now struggled to use the vocabulary of “splitting” and “chunking” to explicitly describe what they were doing. When students “chunked” their snap cubes together they only referred to this as “putting them together” and did not exhibit an explicit awareness that they were using the chunking strategy. In lesson seven we used the snap cubes again and focused on vocabulary to prompt students to use splitting and chunking vocabulary more explicitly to see if they would directly connect their actions to these strategies. We discussed with the students what the splitting and chunking were actually doing to the numerator and denominator of a fraction. For example, the students realized that when chunking snap cubes together they were actually creating equal-size groups of cubes which is what allowed them to find equivalent fractions. Students then used the snap cubes to create their own fractions and find subsequent equivalent fractions.

In these last three lessons, we noticed that students at first struggled with creating equivalent fractions that could not be simplified by dividing numerator and denominator by 2. For example, we found that our students had difficulty recognizing that 3/12 and 1/4 are equivalent because 1 is not half of 3 and 4 is not half of 12. When students worked with a snap-cube representation of a fraction with an odd denominator, they realized they were unable to “double” the denominator by using the splitting strategy. This forced students out of their comfort zones and directed them toward exploring chunking or splitting fractions into groups of more than just two. For example, when given snap cubes that representing the fraction 6/9, students found out that they couldn't split their cubes into equal groups using two and had to explore how they could make equal-size groups. Students were able to split their cubes into groups of three to create the fraction 2/3.

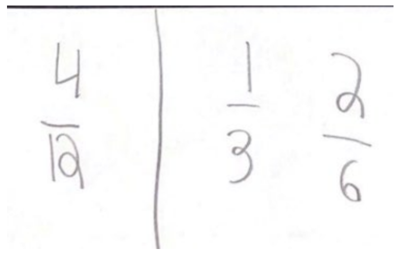

Figure 7. The original fraction a set of snap cubes represented and two student-generated equivalent fractions.

Summary/Discussion

It should be acknowledged that our study has some limitations. We only worked with four students, and our summer program only lasted 10 weeks. Teachers may still find our work valuable, however, as examining the thinking of our students and how it developed over the course of the lessons may help them generate ideas for their own classrooms.

Our students' work over the summer showed how their conceptual understanding of equivalent fractions developed. We found that the use of splitting and chunking resulted in students relying on the strategy of doubling or halving the numerator and denominator of given fractions. However, the use of discrete manipulatives helped our students move away from their reliance on doubling to make an equivalent fraction. Introducing students to fractions with odd numerators and denominators required them to explore other ways of making equal groups and allowed them to independently learn and develop an understanding of different ways to find equivalent fractions. Overall, our students demonstrated improved conceptual understandings of creating equivalent fractions. We hope that our findings allow other teachers to critically think about how to develop students' conceptual understanding of equivalent fractions.

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Differentiation through Small-Group Instruction and Learning Centers

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Valerie Johse, Pearland Independent School District

Abstract: *This article describes elements of our two-year professional development program aimed at helping elementary teachers implement math stations to allow time for focused instruction in small-groups. Teachers from 11 elementary schools met quarterly to receive support for setting up management routines, student expectations, and accountability techniques as well as sample stations aligned to the scope and sequence. We share suggestions for integrating stations into the math period, building students' independence, and differentiating within stations. We offer recommendations for purposeful composition of small groups and tips for reteaching in a small-group setting.*

One-size-fits-all, whole-group approaches are largely ineffective in differentiating for today's diverse mathematics learners. The need for differentiation may be acutely felt by teachers adjusting to teaching post-pandemic. Small group instruction provides focused experiences, tailored challenges, and ongoing support (Copple & Bredekamp, 2009). Groups of students working together experience cognitive and social-emotional benefits including improved communication and problem-solving skills (Slavin, 2014), lengthened attention to a task (Godwin et al., 2016), and acceptance of multiple approaches to problem solving (Kilic et al., 2010). Still, using small groups to differentiate math instruction has yet to take hold in many classrooms (Small, 2020). To promote differentiation in our district's elementary school classrooms, we instituted professional development for student-led stations and teacher-led instruction in small groups.

Professional Development to Improve Differentiation

Our aim was to help teachers successfully implement math stations, freeing them up to provide focused, differentiated instruction with three to six children in small-group instruction. The 11 elementary school principals in our district selected teachers from each grade level to attend our two-year program. We organized quarterly three-hour sessions by grade levels (prekindergarten and kindergarten; grades 1 and 2; grades 3 and 4).

First year objectives of the program included:

1. Introducing math stations
2. Giving research-based support for stations
3. Offering practical implementation suggestions and motivational support for stations
4. Promoting small-group instruction as a form of differentiation

In the second year, we offered:

1. A collaborative environment where teachers could reflect and share their experiences
2. Suggestions for practical challenges such as classroom management and routines
3. Stations like those shown in Table 1 to support the scope and sequence

Table 1. *Sample math stations with links to videos demonstrating each task.*

Activity Name and Link	**Grade**	**Instructions**
Number Stations		
Domino Parking Lot	PreK-1	Take turns picking a domino, adding the pips, and placing it on the correct parking spot. The winner has the tallest stack on any parking spot after all the dominoes have been used. Mats available at www.carriecutler.com
Addition Double Ten Frame War	2-3	Deal ten frames so players cannot see them. Each player turns over their top two cards and adds them. The player with the highest sum takes all the cards in play. The winner is the player with the most cards when all cards have been played.
How Long? How Many?	3-4	Each player has a 10x10 cm. grid. Roll a die to find *How Long* a Cuisenaire rod to take then roll to find *How Many* rods to take. Players place the rods on grid in a rectangle formation, trace, and label with multiplication equation. When a rectangle cannot be placed, count the uncovered units. Player with the lowest score wins.
Geometry Stations		
Straw Triangles	PreK-K	Students form different triangles by cutting straws different lengths and using playdough for vertices.
Tabletop Composing Shapes	K-2	Use masking tape to make a shape on a tabletop. Students use pattern blocks to compose/fill in the shape.
Who's in My Geoboard Club?	3-4	Leader makes a secret shape on geoboard. Players make shapes then show them to the leader who tells them whether or not they are in the club. Players compare their shape, discuss and adjust attributes until they are admitted to the club.
Measurement and Data Stations		
Primary Color Cuisenaire Rod Trains	PreK-1	Take turns rolling a red/yellow/blue die and traditional die. If you roll 4 and red, take 4 red rods and make a train by placing them end-to-end. The player with the longest train after three turns wins. Next time play shortest train.
Capacity Comparisons	1-2	Predict order from least to greatest capacity of variety of containers. Measure and report findings. May use other attributes: area, perimeter, length, weight, volume, etc.
Which is Quickest?	3-4	Students estimate time needed to complete activities such as blink eyes 5 times, use a spoon to fill a cup with beans,

		complete a puzzle, etc. Use a stop watch to time how long it takes to do each activity.
Algebra & Patterning Stations		
Pattern Towers	PreK-1	Students poke a chopstick into a mound of dough then slide on 1" straw colored pieces to build vertical patterns.
Hundred Chart Arrow Problems	2-4	An arrow problem uses only arrows as clues to find a secret number on a hundred chart. Begin with the starting number and move one number, one row or one column on the hundred chart for each arrow clue. Students write a series of clues and try to stump each other.

We also suggested long-term stations that could be used throughout the school year to support big ideas in math such as number sense, real-world connections, and mathematical literacy. An estimation station, computer or board games, and math-themed picture books reduce teacher preparation and provide variety.

Implementing Math Stations

Students must work independently in stations to maximize a teacher's uninterrupted small-group instruction time. Our teachers needed support in creating classroom environments that addressed the physical and social-emotional aspects of small-group learning. This Math Stations Checklist promotes students' self-directed work in stations:

- ☐ Are supplies organized in tubs or bags that can be taken from a central area to assigned locations?
- ☐ Are specific directions for the activity displayed at the station or in the materials tub, including sample activities when appropriate?
- ☐ Is the management system clear and consistent?
- ☐ Do students know which station to use and how much time is allotted?
- ☐ Do students know which activities are to be done individually, with partners, or in small groups?
- ☐ Has each station been introduced to the class, including clear expectations on:
    - Caring for materials?
    - Cleaning up after use?
    - Asking for assistance?
- ☐ Are work product requirements clear, including recording sheets or math journal entries?
- ☐ What are the expectations for behavior in stations?

Management systems. We suggested two approaches for scheduling students for stations—a rotation system or a student-selected system. In a rotation system, students systematically move through stations according to a chart or wheel indicating station assignments. In a student-selected system, students choose which stations they visit and the teacher may or may not impose time constraints. In both systems, limiting the number of students in a station encourages on-task behavior.

*Giving choices. S*tudents reveal their mathematical strengths, preferences, learning styles, and degree of responsibility when teachers allow some freedom to choose activities during station time. The teacher may designate certain stations as mandatory while others are optional. Regularly conferencing with students allows the teacher to guide them where they *need* to go as well as where they *want* to go.

Station expectations. Students feel greater ownership over the experience when they help define routines and behaviors. Discuss procedures in detail and role play how to respond to signals for cleanup. Emphasize that "up and around" does not mean "loud and out of control."

Number of stations. The number of stations can vary depending on how comfortable a teacher is with simultaneous activity in the classroom. Students may revisit the same station periodically to attempt more sophisticated approaches and celebrate growing competency with concepts.

Grouping students. Student groups need not and should not remain stagnant. Grouping students heterogeneously allows for peer learning and scaffolding. Occasionally, a teacher may choose to differentiate instructional tasks based on a specific mathematical need by assigning students with similar abilities to work together at an appropriately challenging station.

Implementing stations. We created a six-week plan to help teachers gradually implement stations. In the first two weeks, teachers introduce long-term stations, math journals, and recording sheets for accountability and begin establishing routines and expectations. During weeks 3 and 4, teachers present a new station every few days, modeling the station with two students while the class watches. Those two students teach two more students—each one teach one—until all students learn the activity. During week 5, students visit one station a day for about 15-20 minutes with time allowed for practicing locating materials, engaging in the station, following help procedures, and cleaning up. By week 6, students can move through two or more stations per day following the management system, allowing the teacher to initiate small-group instruction. Teachers might divide their 90-minute math period to include:

- 10-15 minutes for warm up or spiral review
- 40-50 minutes for whole group instruction (should contain elements of small-group and partner work)
- 30 minutes for math stations and small-group instruction with the teacher
- 5 minutes for reflection and sharing

Practicing help procedures. Students need consistent routines for obtaining help when the teacher is occupied with a small group. One of our teachers wore a crown when leading a small group. When students saw the crown, they did not interrupt. Students who need help can "ask 3 then me" or set a tent-folded card labeled with a question mark on the desk and the teacher can provide assistance at the conclusion of the small group.

Individual accountability. On-task behavior and personal accountability increase when students view stations as integral to their learning. Students build organizational skills by maintaining their own records of math station work, including:

- Activity menus where students check off completed stations

- Journals where students use words, pictures, or other means of documenting learning
- Station folders for each student with general recording sheets stapled on one side and projects and papers placed in a pocket. The teacher checks folders regularly.

Differentiating within stations. Groups of students may be working on different stations around the classroom, but differentiating materials *within* a station also provides individualization as students of varying mathematical competencies work together. Supplying dice or spinners with ranges of numbers can instantly alter the difficulty of a task. Color coding can guide students to the suitable level of materials. For example, place card-based games in baggies labeled with a colored dot to indicate degree of difficulty. A teacher might differentiate with parallel tasks designed to meet the needs of students at different developmental levels (Small, 2020). Students choose or teachers can assign one of two or three similar tasks that address the same learning goal and are close enough in context that they can be discussed simultaneously by the students in the station. For example, two students may complete Option One: *Use 20 toothpicks to make three shapes. None of the shapes can use the same number of toothpicks. Describe your shapes*. Two other students in the station might complete Option Two: *Use 20 toothpicks to make at least four shapes. Describe your shapes.* Both tasks address similar mathematics ideas while allowing for appropriate challenge.

Reflecting on learning. At the end of each math-station period, allow time for students to discuss or write in math journals about their station experiences, math strategies, and solutions. Reflection helps students to articulate their understanding and move mathematical ideas from short- to long-term memory.

Teacher-Led Small Group Instruction

Working with a small group of students allows the teacher to monitor understandings and attitudes as well as strengths, weakness, and preferences in mathematics. The teacher carefully selects students with whom (s)he will meet during this powerful learning time.

Using small group instruction to differentiate. Every student desires the teacher's attention; therefore, every student should receive some time in small-group instruction. The frequency a teacher meets with a student depends on his/her needs; however, we encouraged our teachers to set a goal to meet with each student at least twice a month. During small groups, the teacher can match teaching strategies to students' needs by using a variety of manipulatives, activities, and techniques.

Using small-group instruction for reteaching. We found that working with struggling students in a focused setting resulted in improved student understanding. Some of the reasons we believe this approach to be effective are:

- Targeted instruction at the student's level
- Step-by-step guidance from the teacher
- Close monitoring of manipulatives use
- Immediate corrective and/or positive feedback from the teacher
- Increased mathematical communication, particularly from hesitant learners
- Decreased peer pressure

Below, we offer five important ideas to get the most out of reteaching in the small-group setting.

1. Assess. Use some form of assessment to determine which students need reteaching. The assessment should come prior to the end-of-unit assessment and may be observational rather than traditional paper and pencil.

2. Assemble. Carefully select a small group of students for whom the mathematical content seems unclear.

3. Concrete Approaches and Questioning. Look for gaps in understanding by carefully observing the children *doing* mathematics. Start at the most *concrete* forms of representation to reteach the concept. Move on to pictorial, then to the abstract. Many students need repeated exposure to multiple sense-making, hands-on tasks before they can assimilate a concept. Use questioning and scaffolding to guide students' understanding. Remember, *telling* students which solution is correct is never as powerful as letting them *figure it out for themselves*, even if the latter does take more time.

4. Reassess. Determine if concepts have been clarified. Use developmentally-appropriate forms of assessment. Consider students' preferred forms of expressing mathematical knowledge.

5. Return. Avoid consistently pulling the same students for teacher-led small group instruction. Return students to other small-group experiences where they can learn from peers, develop relationships, and build math-positive mindsets.

Feedback from Teachers

Teachers reported that interactions at stations and personalized attention offered during small-group instruction led to a more equitable mathematics environment. They appreciated focused time with small groups where they could watch as students engaged in mathematics rather than collecting a product that may not reveal students' strategies. Recognizing students' preferred approaches, strengths, and weaknesses helped teachers plan future differentiated lessons and activities. Teachers also admitted that the implementation process was not without occasional roadblocks, particularly when students struggled to work in stations without close supervision. They addressed this by adjusting stations to make them more engaging and by reinforcing on-task behavior. Teachers responded positively to our professional development program, saying:

- "I've been doing too much. I should have quality stations that kids enjoy and that reinforce their learning, but I don't have to change them every day."
- "Oh, so in math everyone doesn't do everything every day. I've been killing myself trying to do it all wrong."
- "This all makes sense now. I'm going to try it."

Conclusion

Modern classroom diversity demands equally diverse teaching methods. Post-pandemic classrooms must address gaps in understanding and provide appropriate challenges to help students move ahead on their mathematics journeys. Differentiation is not more of the same approaches but carefully selected tasks and strategies for helping students understand mathematics deeply and thoroughly. Use of stations and small-group instruction empowers teachers to meet the differentiation needs of their widely varied students and provides support for all learners.

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Valerie Johse has been a teacher and district elementary math specialist as well as a curriculum writer and consultant for educational publishers. She is dedicated to improving learning outcomes for students from all backgrounds by providing high-quality professional development and support for teachers.

Deal-o-rama: Expected Value in Unexpected Places

Ryan D. Fox, Belmont University

Abstract: *When is a deal a good deal? Especially involving games of chance, when does a game become a fair game to a player? Follow along as I present an exploration of weighted averages, or expected value in statistics, to determine fairness in the context of a new game: one that combines mathematical ideas from two classic television game shows. An activity for students to explore this concept is provided.*

Televised game shows provide fascinating mathematical explorations by connecting mathematical concepts to practical applications. Researchers determined the fairness of deals offered to contestants in the game show *Deal or No Deal* (Post, van den Assem, Baltussen, & Thaler, 2008). Marilyn vos Savant, in a famous column in *Parade* magazine (Pond, 2016), discussed strategies to winning prizes located behind curtains in the game show *Let's Make A Deal*. I present an activity that synthesizes these two games.

In Appendix A, I present a new game called *Deal-o-Rama*. It combines elements of two game shows. This game addresses multiple high school standards from the Common Core (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010):

Table 1. *Mathematics standards address in this activity*

MD.A.1. Define a random variable for a quantity of interest by assigning a numerical value to each event in the sample space. MD.A.2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. MD.B.5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. MD.B.5a. Find the expected payoff for a game of chance. MD.B.5b. Evaluate and compare strategies on the basis of expected values.

I can express the new game in the following stem:

> The host of *Deal-o-Rama* shows a contestant ___ closed doors. Behind one of the doors is a valuable prize worth $___; behind the remaining doors is nothing. The host opens a door not containing the prize. After the reveal, the host offers $___ as a lure to stop playing the game. If the contestant accepts the offer, the contestant cannot win the grand prize. What

is the contestant's best strategy at this moment: keep playing to win the big prize or stop playing to collect the monetary offer?

Elements of the two games appear in this example. In *Let's Make A Deal*, a contestant chooses one out of three doors. Of the three doors, one door hides a valuable prize while two doors do not hide prizes. The contestant's goal is to choose the prize-hiding door. After the contestant selects one door, the host reveals one of the two non-winning doors and offers the contestant the opportunity to keep the originally selected door or to switch to the third door. The host has knowledge of the winning and non-winning doors. *Deal or No Deal* involves a selection of briefcases containing various cash prizes. The contestant's goal is to select the briefcase containing the largest amount of cash. Once the contestant selects a briefcase, the contestant selects remaining cases to reveal other cash prizes from the game. At specified intervals, the host of the show offers the contestant a deal, an amount of money for the contestant to win instead of continuing the game. The deal offered to the contestant is not dependent on the contestant's selection, but instead based on the expected value of all the remaining unrevealed selections. In this game, the host does not need knowledge of the specific location of all prizes to make an acceptable deal to the contestant.

In the activity in Appendix A, I combine key aspects of each of the two games in a different way. In the activity, like both game shows, a contestant makes an initial selection. Like *Let's Make A Deal*, a host—and not the contestant—will select a non-winning door; the host knows that the revealed door is a non-winning door. Like both shows, the host asks the contestant to decide on a deal. Like *Deal or No Deal*, the host makes an offer to the contestant based on the expected value of the remaining doors. The offer and the subsequent determination of its fairness could be judged against the expected value of winnings of those doors. Students playing the activity in Appendix A determine for themselves the best strategy to use.

I will illustrate a six-door example of *Deal-o-Rama*, beginning with Figure 1. This example extends the two situations mentioned in Appendix A. I will return to those activities later. For this extended example, I will make the value of the prize $30,000. A contestant chooses any of the six labeled doors, hoping to win the $30,000 prize.

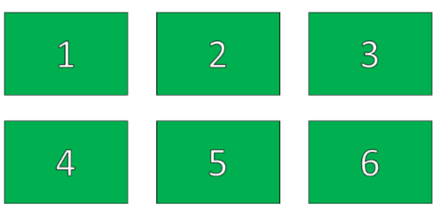

Figure 1. The set-up for the example *Deal-o-rama.*

In this example, the contestant selects door 3, as illustrated in Figure 2. Addressing Standard MD.A.1 from Table 1, the probability the contestant chooses the winning door in this example is 1/6: any door is equally likely to be the winning door as the other five. This choice also suggests the probability of the chosen door being the non-winning door as 5/6. Addressing Standard MD.A.2, the expected value of an individual door is $\frac{1}{6} \times 30000 = 5000$. In this example, the average value of one door right now is $5000.

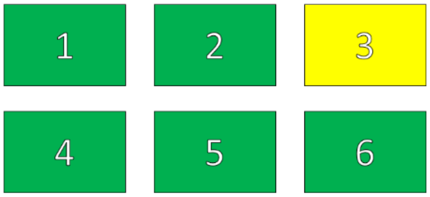

Figure 2. The contestant's selection in the example game of *Deal-o-rama.*

The host, with the knowledge of the location of the winning prize, reveals door 4 as illustrated in Figure 3 with an X.

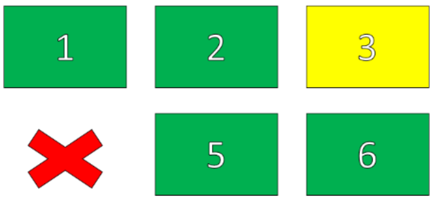

Figure 3. The selected and one revealed door.

At this point we reach the first decision point while addressing the Common Core Standards in Table 1. The combined probabilities of doors 1, 2, 5, and 6 being the winning selection is 5/6, equaling the probability a door other than door 3 is the winning door. Reflecting on Standard MD.A.1, notice the probabilities for the four unrevealed doors are not the same as five unrevealed doors. We must assign different probabilities to each of the four doors with this new information. Because each of the remaining unselected doors (1, 2, 5, and 6) are equally likely to be the winner as the others right now, each unrevealed door has a $\frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$ probability of being the winning selection. Looking at Standard MD.B.5a with the prize valued at $30,000, the expected value of winning for doors 1, 2, 5, or 6 is $\frac{5}{24} \times 30000 = 6250$. The expected value of winning of the originally selected door remains $5,000, but the expected value of the winning of the four unrevealed doors is now $6,250. Addressing Standard MD.B.5b, an offer from the host to the contestant greater than $6,250 is more valuable than any unrevealed door. This offer would be fair to the contestant and worthy of consideration to stop the game and claim the offer. Any offer less than $5,000 should be rejected by the contestant, because the expected payoff of any door is greater than the value of the offer.

Should the contestant continue to play, the host would remove another non-winning door as seen in Figure 4. Returning to Standard MD.A.1 each of the three unrevealed doors containing the prize now has a probability of containing the prize of $\frac{5}{6} \times \frac{1}{3} = \frac{5}{18}$. The combined probabilities of a door other than the contestant's selection—probability of 5/6—is evenly divided among the last three unselected doors: 2, 5, and 6. Using wording from Standard MD.B.5a, the expected payoff of each of the unselected doors is $\frac{5}{18} \times 30000 = 8333$ when rounded to the nearest dollar. Applying Standard MD.B.5b to the host's offer, the contestant should consider accepting any offer that is worth more than $8,333. Any offer more than $8,333 is more valuable than any door in the game. As before, any offer less than $5,000 should be rejected because the offer is less than the expected payoff of any door. If the contestant chooses the offer, the game ends with the contestant claiming the offer. Should the contestant not accept the offer, the game continues with the contestant facing more choices.

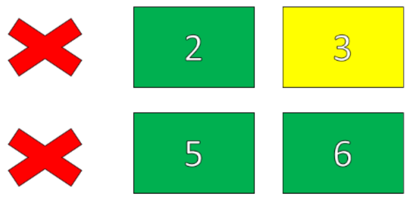

Figure 4. Host reveals another non-winning selection.

In Figure 5, the host reveals and removes another non-winning selection. Each unrevealed door now has a probability of containing the grand prize of $\frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$ and a corresponding expected value of winning of $\frac{5}{12} \times 30000 = 12500$. The decision remains the same: compare the value of the host's offer to the expected value of the original selection, still at \$5000, or the remaining unopened selections, each at \$12,500.

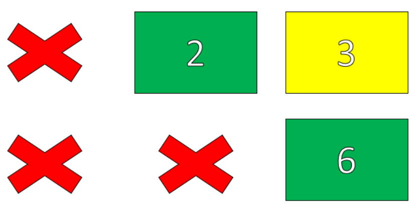

Figure 5. The contestant's final deal: only two selections remain unseen.

For this example, we will reach the last decision with the reveal of a fourth non-winning door as seen in Figure 6. Now the host can present three options: keep the original selection, receive an offer to end the game, or select the last remaining door (door 2 in this example). At this point, the most likely consideration would be to select the last unselected door, unless the host presents an offer greater than $\frac{5}{6} \times 30000 = 25000$.

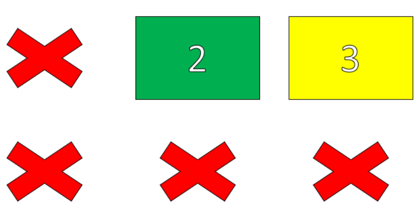

Figure 6. The final decision of the six-door game.

We return to the four- and five-door cases in the Appendix. I can show how the offer compares to the expected value and what is seemingly the best decision in order to address Standards MD.B.5, 5a, and 5b. Starting with four doors and a \$10,000 prize, the expected value of each door initially is \$2,500 as $2500 = \frac{1}{4} \times 10000$. The contestant should not accept the \$2,000 offer in Question 1a as the offer is less than any door's expected payoff. The host's offer in Question 1b is a substantial improvement over the offer in Question 1a. However, with two doors removed, only the contestant's original selected and one unrevealed door remain. The expected payoff of the originally selected door remains at \$2,500, while the expected payoff of the last unrevealed door is \$7,500. The offer is in between these two values. The decision is for the contestant to make. Simulating the activity could support or refute the contestant's—or student's—decision. The contestant, if given the opportunity, should swap doors as described in Pond (2016) or take the offer if not given an opportunity to swap selections.

In Round 2 of the activity in the Appendix, we start with five doors and a \$20,000 prize. The expected value of a single door in this round is \$4,000. As in Round 1, the offer in Question 2a—\$3,000—is far too low compared to any expected value: \$4,000 for the selected door and \$5,333 for the remaining doors. The expected payoff is the dividing of the probability of winning across three doors, $\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$, multiplied by the value of the prize, $\frac{4}{15} \times 20000 = 5333$ when rounded to the nearest dollar. Question 2b offers an interesting consideration: the value of the offer is better than the expected payoff of any door, the original door at \$4,000 and the unopened doors at \$8,000. As the best value among any option, the contestant should consider accepting the offer. Question 2c offers a different consideration for the contestant as the difference between the host's offer of \$15,000 is not much less than the expected value of the unselected door at \$16,000. Accepting the offer or switching doors is a better decision than keeping the original selection. Once again, let the students practice this activity to determine their best strategy.

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Appendix A: The Activity, *Deal-o-Rama*

In this activity, you are a contestant on a game show called *Deal-o-Rama*. In this game, you must decide what is the best strategy to win the most valuable possible prize.

Round 1. The host of *Deal-o-Rama* shows you 4 closed doors. Behind one of the doors is a valuable prize worth $10,000; behind the remaining doors is nothing.

Question 1a. After you choose a door, the host opens a door not containing the prize. After the reveal, the host offers $2,000 as a lure to stop playing the game. If you accept the offer, you cannot win the grand prize. What is the best strategy at this moment: keep playing to win the big prize or stop playing to collect the offer? Why did you choose that strategy?

Question 1b. The host opens a second door not containing the prize. The host offers $5,000 as a lure to stop playing the game. If you accept the offer, you cannot win the grand prize. What is the best strategy at this moment: keep playing to win the big prize, stop playing to collect the offer, or switch doors to the final door to win the prize? Why did you choose that strategy?

Round 2. The host of *Deal-o-Rama* shows you a second set of 5 closed doors. Behind one of the doors is a valuable prize worth $20,000; behind the four remaining doors is nothing. With the extra challenge comes a larger prize.

Question 2a. After you choose a door, the host opens a door not containing the prize. After the reveal, the host offers $3,000 as a lure to stop playing the game. If you accept the offer, you cannot win the grand prize. What is the best strategy at this moment: keep playing to win the big prize or stop playing to collect the offer? Why did you choose that strategy?

Question 2b. The host opens a second door not containing the prize. After the reveal, the host offers $9,000 as a lure to stop playing the game. What is the best strategy at this moment: keep playing to win the big prize or stop playing to collect the offer? Why did you choose that strategy?

Question 2c. The host opens a third door not containing the prize. After the reveal, the host offers $15,000 as a lure to stop playing the game. What is the best strategy at this moment: keep playing to win the big prize, stop playing to collect the offer, or switch doors to the final door to win the prize? Why did you choose that strategy?

Preservice Teachers' Reflections on Teaching for Equity During Ongoing Racial Injustice

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Abstract: *Elementary preservice teachers (PSTs) completed an assignment where they reacted to leading mathematics education professional organizations' statements on recent events involving racial injustice in the United States in the context of a larger assignment where they reflected on what it means to teach mathematics for equity. Results show that when presented with the organizations' statements, PSTs were more hesitant to engage with teaching mathematics for equity than students in a previous study who had not been presented with such statements. Implications of this disparity are discussed.*

Introduction

Teaching mathematics for equity and social justice is of critical importance for teachers and students across the PK-16 spectrum. Often teachers and preservice teachers (PSTs) have trouble conceptualizing just how to do that, though (Jackson & Jong, 2017), despite the consistent calls to do so from major mathematics education organizations. The study reported on in this article sought to understand what current PSTs understood and took away from two major mathematics education organizations' publicized statements regarding racial injustices in the United States (NCTM, 2020; AMTE, 2020) after 1) an initial reading of the statements, 2) completion of a series of reading and reflecting activities using articles on equity and culture in mathematics education, and 3) rereading the organizations' statements as well as their original reflections and then re-reflecting.

This study partially replicates and builds upon a study by Jackson and Jong (2017) titled *Reading and Reflecting: Elementary Preservice Teachers' Conceptions about Teaching Mathematics for Equity*. In Jackson and Jong's study, the participants were elementary PSTs in a mathematics methods course the semester before they student taught. The authors had the PSTs read two works: Secada and Berman's (1999) chapter *Equity as a Value-Added Dimension in Teaching for Understanding in School Mathematics*, and McCulloch, Marshall, and DeCuir-Gunby's (2009) article *Cultural Capital in Children's Number Representations*. After reading the articles, they asked the PSTs to submit written reflections on three questions:

1. Read the *Equity* chapter. Reflect upon the following: The authors argue that teaching for understanding promotes greater equity. Do you agree or disagree with this argument? Why or why not? Please include a classroom example to support your response.

2. Read the *Cultural Capital* article. Reflect upon the following: In your classroom, what do you foresee as potential benefits and challenges of incorporating students' cultural and diverse backgrounds when teaching mathematics?
3. What new thoughts/ideas/questions did the readings raise for you in regards to teaching mathematics? OR In what ways did the readings build upon existing ideas you had about teaching mathematics?
   (Jackson & Jong, 2017, p. 70)

Jackson and Jong used these readings and reflection questions "to learn about PSTs' conceptions of equity in relation to teaching for understanding and the role culture has in mathematics instruction" (p. 70). After the PSTs submitted their written reflections, the authors engaged the class in a discussion about what they had written.

In discussing their analysis of the PSTs written reflections, Jackson and Jong (2017) reported that most PSTs recognized the importance of broadening their perspectives and making the mathematics they teach relevant to their future students. A significant minority of the PSTs, though, felt it was important to just focus on mathematics and not students' cultures. They reasoned, among other things, that addressing culture was inappropriate in a mathematics class, it would take a lot of time to do correctly, they may not be able to do it adequately, and they might receive negative feedback from parents and other sources. After the class discussion, Jackson and Jong noted that "While not all PSTs agreed with the perspectives discussed in the assigned readings, … [and] we do not believe the conceptions of all PSTs were changed," (p. 77) that the activity did at least expose all the PSTs to these viewpoints, and the PSTs might critically reflect on them in the future.

Participants

The present study consisted of eight elementary PSTs enrolled in a mathematics content course during June 2020 at a large regional university in the Mid-South. All eight of the students were female. Two students were African American, and the rest were white. Given that the assignment described below involved reading and reacting to statements made by major mathematics education organizations regarding the recent murders of African Americans, the African American students were notified of the nature of the assignment ahead of time and given the option to do a different assignment for the same amount of credit. Research has shown that "police killings of unarmed black Americans have adverse effects on mental health among black American adults in the general population" (Bor et al., 2018). Because of this, faculty need to take care to not retraumatize students. One way to accomplish this is to provide African American students with advance notice of the assignment topic and provide an alternative assignment if the students desire it (Mitchem, 2021). In this case, both students chose to do the assignment described below.

The participants in this study were also at an earlier point in their PST education programs than the participants in Jackson and Jong's (2017) original study. Jackson and Jong's students were in a mathematics methods course that included practicum experiences in diverse schools the semester before they student taught. The PSTs in this study were enrolled in the first of three mathematics content courses that are taken before their methods course, which in turn is taken before student teaching. So, as a reference for comparison, these PSTs were all at least three semesters from student teaching. There were no practicum experiences associated with this content course.

Methods and Data Collection

The present study sought to understand what PSTs understood and took away from the National Council of Teachers of Mathematics' (NCTM's) and the Association of Mathematics Teacher Educators' (AMTE's) June 2020 statements on the deaths of George Floyd, Breonna Taylor, and Ahmaud Arbery. The assignment consisted of five parts, which are summarized in Table 1 below. In Part I, the PSTs were given two brief paragraphs describing the missions of NCTM and AMTE and the two statements mentioned above. The PSTs were asked to write a paragraph summarizing the NCTM statement, a paragraph summarizing the AMTE statement, and to respond to the following prompt: "Considering your role as someone who is preparing to become and will soon be a teacher, write at least half a page in reaction to these two statements. The reaction is yours to write as you wish, but feel from to include any questions or uncertainties you may have." In Part II, the PSTs were given Secada and Berman's article (1999) and asked to write a reflection on Question 1 from Jackson and Jong (2017). In Part III, the PSTs were given McCulloch, Marshall, and DeCuir-Gunby's article (2009) and asked to write a reflection on Question 2 from Jackson and Jong (2017). In Part IV, the PSTs were given Jackson and Jong's article (2017) and asked to write a reflection on whether they identified with any of the viewpoints raised by the PSTs in the study or those of the authors. In Part V, the PSTs were asked to reread the NCTM and AMTE statements, reread what they submitted for Part I, and asked to respond to the following prompt: "Having completed the other parts of this assignment, is there anything you would like to add/update/change/reiterate about your original submission for Part I? Why or why not?" Unfortunately, due to time constraints, a class discussion of this assignment did not happen as happened in Jackson and Jong.

Table 1. *Description of the Assignment*

Part	Description
Part I	Read NCTM's and AMTE's statements on systemic racism, summarize them, and write a reaction to them considering your future role as a teacher.
Part II	Read Secada and Berman (1999), and write a reflection on whether you agree or disagree that teaching for understanding promotes greater equity.
Part III	Read McCulloch, Marshall, and DeCuir-Gunby (2009), and write a reflection on what you see as the potential benefits and challenges of incorporating students' cultural and diverse backgrounds when teaching mathematics.
Part IV	Read Jackson and Jong (2017), and write a reflection on whether you identify with any of the viewpoints of the PSTs in that article or the authors' viewpoints.
Part V	Reread NCTM's and AMTE's statements, reread what you submitted for Part I, and address whether you would like to add/update/change/reiterate anything about your submission for Part I.

Results

One of the main findings of this study was that the PSTs collectively made very few revisions to their Part I submissions in Part V. While reading the other articles may have been helpful to the PSTs in other ways, they appear to have had little effect on the PSTs' reactions to the organizations' statements.

Compared to Jackson and Jong's (2017) results, the present study's results varied more in type and had a larger percentage of students (50%) expressing uncertainty or hesitation about addressing students' cultures in their future classrooms. While the smaller number of participants in this study means each individual response has a greater effect on the results, this shift toward uncertainty could potentially be the result of being presented with the organizations' statements on a series of very current and emotional events. For example, one representative student stated in Part I and reiterated in Part V that "I wouldn't want to express my opinions as it might influence their thinking and upset their parents causing even more problems. It creates the risk that my students may tell their parents that I am talking about social issues and create more problems within our society."

Two other types of responses showed up in this study that Jackson and Jong (2017) did not report finding in their study. The first of these was an overtly racist response to the activity by one white PST in her written reflections. For example, as part of her reaction to the organizations' statements in Part I of the assignment, she wrote "I do not think it's fair for any of the families who have lost a family member due to the rioting, or their home, or place of business." She continued to focus on that aspect of the events of June 2020, shifting her focus away from the topic of the organizations' statements and the effects of the current events on potential African American students she might have.

The other type of response found in this study that Jackson and Jong (2017) did not report finding was PST attempts to disengage from the challenge being addressed by the assignment entirely. Jackson and Jong noted that some PSTs did not agree with the viewpoints presented by the assignment they did, but they still engaged with the assignment and discussed their disagreements. In the present study, when asked to summarize the two organizations' statements in Part I, two of the eight PSTs chose to summarize the brief paragraphs I provided them to give them context about the organizations' missions and never addressed the statements made by the organizations. When asked to revisit the organizations' statements in Part V of the assignment, one of these two PSTs again just referred to the context paragraphs and never the organizations' actual statements.

Discussion

Teaching mathematics for equity and social justice is of critical importance for teachers and students across the PK-16 spectrum. As Jackson and Jong (2017) showed, some PSTs are hesitant to do this and think it may be inappropriate. The present study superimposed PSTs reading and reacting to unequivocal statements about racial justice and current events by NCTM and AMTE on top of the procedure conducted by Jackson and Jong. Although the sample size was small, the proportion of students expressing hesitancy or uncertainty about addressing social justice topics in their classrooms increased. Responses that showed disengagement and overt racism were also present. For these reasons, it was unfortunate there was not a class discussion of the assignment. Such a discussion may have helped to mitigate the higher levels of hesitancy and other negative responses in the written reflections.

Part of the need to teach mathematics for equity is to consider everything students bring with them into the classroom in terms of funds of mathematical knowledge as well as individual and cultural backgrounds, all of which overlap. We as teachers, including PSTs, also need to critically consider everything we bring with us into the classroom if we are to teach equitably. If a tragic event or series of events occurs in our nation, and our leading mathematics education professional organizations put out forceful statements in defense of our students, and yet our individual reaction is to become more hesitant to engage in equitable math teaching for social justice, we have to critically examine our practice and ask ourselves why that is.

With this in mind, statements like those made by NCTM and AMTE in June 2020 are more than mere press releases on behalf of the organizations; they can and should be powerful teaching tools. In my role as a mathematics educator for elementary PSTs, I was able to use the statements to draw out thoughts and reactions from the PSTs that may well have gone unnoticed if they had just completed Parts II-IV of the assignment. Once those thoughts and reactions are noticed, they can be proactively addressed. Because of their ability to draw out these reactions, the organizations' statements can also be powerful tools for practicing teachers as well, even if they have not read the other articles included in the assignment in this study. The statements can be read and discussed among colleagues. As in Jackson and Jong's study of PSTs, all practicing teachers may not agree with all aspects of the viewpoints expressed; however, being exposed to the organizations' statements and colleagues' thoughts about them can help us make positive progress with respect to equitable mathematics teaching by showing us where we need to do some of the most critical work.

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