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***Banneker Banner* Submission Guidelines**
The Banneker Banner is the official journal of the Maryland Council of Teachers of Mathematics. The journal is named after Benjamin Banneker, a Maryland native and perhaps the first documented African-American mathematician. The Banner is published twice each year and contains a wide range of articles on issues in mathematics education at all levels. Articles published in the journal must be submitted to the editor and undergo a peer review process.

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. All submissions should be relevant, interesting, and useful to teachers of mathematics in Maryland. More information about the submission process can be found here.

Go with the Flow: Teaching Fractions using Water

Matthew Wells, Montgomery County Public Schools

Abstract: *This article focuses on instruction in grades three and four, the levels at which learning of fractions is formally introduced in the Maryland Common Core State Curriculum and solidified before middle school. It describes a strategy of incorporating water in a hands-on approach to learning fractions. Best practices for its implementation and a justification for its efficacy are included.*

Elementary school teachers are routinely looking for ways to provide innovative and engaging instruction for their students to deepen their understanding of foundational concepts. In this article, I will focus on instruction in grades three and four, the levels at which learning of fractions is formally introduced in the Maryland Common Core State Curriculum and solidified before middle school. As a mathematics intervention teacher in these grades, I have utilized a strategy of incorporating water in a hands-on approach to learning fractions. Throughout this article I will further detail this activity, best practices for its implementation, and a justification for its efficacy.

Students have difficulties learning fractions for several different reasons. Difficulties in performance with whole numbers in the lower elementary grades correlate to lower performance with fractions. Within this cohort of students who struggle with fractions, a major deficit is fraction comparison and equivalency (Hansen, Jordan, & Rodrigues, 2017). Other root causes of struggles with fractions include lower language skills and attention issues. It is for these reasons, I have identified using water to be an effective intervention to teach the skills of comparison, equivalency, and reasoning with fractions.

Of the twenty strategies identified as best practices for English Language Learners (ELLs) in an article in *Intervention in School and Clinic*, at least three of them directly relate to the activity I will describe in this article: teaching vocabulary using realia (objects and materials from everyday life) and demonstration; using manipulatives to make problems concrete; and applying problems to daily life situations. These strategies enable teachers to reach more students than the traditional direct-instruction or paper and pencil drill and practice forms of instruction (Furner, Yahya, & Duffy, 2005). While these strategies were derived to support ELLs, they promote equity for all students who may need extra help with fractions by addressing one of the root causes of their difficulties, accessibility of material for diverse learners.

Another roadblock to mathematical success is anxiety. In her book, *Learning to Love Math*, Judy Willis, a medical doctor, chronicles the impact of mathematics anxiety on a child's brain as well as concrete ways to avoid it. She explains, “Neuroscience research reveals a connection between enjoyable, participatory learning and long-term memory… With interest and lasting memory, your students can learn math with a depth of comprehension that extends beyond the test – and even beyond summer vacation” (Willis, 2010). In addition to sharing many strategies that benefit students experiencing mathematical anxiety, Willis presents a convincing, science-based case for using a variety of sensory inputs, particularly when introducing new learning to students, explaining that, “starting a new topic by presenting ways that students will soon be able to use the knowledge for something they like, such as building something… gives the lower, pleasure-seeking brain motivation to attend to the lesson” (Willis, 2010).

Fractions in upper elementary school are a complex topic that incorporates work done in the primary grades around number sense, measurement and geometry. In order to be successful, students in grades 3-5 need to view fractions as numbers and make connections and applications to work in the lower grades (National Governors Association Center for Best Practices, 2010). For some students, these connections are made organically, but many other students benefit from interventions such as scaffolding that will connect their work in prior grades to their explicit fraction work in grades 3 – 5 as well as hands-on, guided activities to introduce and solidify learning.

One such activity that solidifies learning is creating fractions using a cylindrical cup and water. This lesson can be implemented in many ways, and I will present some lessons learned from my experience, as well as lesson seeds that relate to specific standards. The materials necessary are a plastic cylindrical cup for each participating student, a dry erase marker for each student, a dropper, and a big container of water. It is a good idea to have a towel on hand, in case of the inevitable spills that accompany a fun and engaging activity, or beans could be used instead of water for a cleaner experience. These activities lend themselves to small group instruction, to ensure close supervision, but could also be presented as a whole group demonstration. Transparent, plastic cylindrical cups can be found online or at various stores but, for this activity, it is important that they do not taper towards the bottom. In order to connect this activity to linear and area models of fractions, a cylindrical shape is important.

To introduce this activity, students benefit from some structured time to experiment with the materials. Giving students time to “play” before beginning will yield better focus throughout the lesson. Exploration and engagement with the materials is pivotal for students to draw conclusions from the material, and while teachers possess the underlying mathematical and practical concepts behind given materials, students need to acquire that information during this time (Furner & Worrell, 2017). Let students fill and empty their cup, engage their senses with the water, and ask questions such as, “Can you show me what one whole would look like,” or “Can you show me what zero would look like?” Ask questions that do not have wrong answers such as, “What does the water feel like?” or “Do you like to drink water?” Even simple conversation about water will lower their anxiety. At this point, students will be engaged in the activity and can engage in any of the following lesson seeds. The activities below are by no means exhaustive, and teachers should feel free to adapt, improve and expand them to meet the needs of their students:

Composing Fractions Using Water

Connection to Maryland College and Career Ready Curriculum Framework:

- *3.NF.A.1 - Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.*
- *Knowledge of the relationship between the number of equal shares and the size of the share (1.G.A.3)*
- *Knowledge that unit fractions represent 1 of the total number of parts, for example, the fraction is formed by 1 part of a whole which is divided into 4 equal parts*
- *Knowledge of the terms numerator (the number of parts being counted) and denominator (the total number of equal parts in the whole)*
- *Knowledge of the size or quantity of the original whole when working with fractional parts*

Discussion questions:

- If a fraction is partitioned into X equal parts, what does each part represent?
- What happens to the size of a unit fraction as the denominator increases/decreases?
- What is the smallest possible fraction?
- What is the largest possible fraction?

Steps:

1. Working in partners or small groups, students partition or break apart the water to make different unit fractions. Since grade 3 expectations are denominators of 2, 3, 4, 6, and 8, give each group either 2, 3, 4, 6, or 8 cups. Fill one of the cups in each group with water. The total number of cups will be the denominator.
2. Ask students to pour their water so that there is an equal amount of water in each cup. Once each group is done, have them present to their peers.
3. Encourage students to compare the amounts of water in the cups. In order to facilitate a discussion about the value of each fraction, ask, "Which fraction or cup would you want to have on a hot summer day, why?" To promote higher-order thinking, pour less water in a cup than the 1/8 group's cup and ask, "What fraction could be represented by this cup?"
4. Keep getting smaller and smaller until students can conclude that if the numerator is the same, a larger denominator creates a fraction that is smaller, eventually presenting a cup with only a drop of water in it.
5. Have students compete to identify the smallest unit fraction, writing the fractions down so that they can make use of repeated reasoning, *Standard for Mathematical Practice Eight* (National Governors Association Center for Best Practices, 2010). This activity can be presented later, having students identify the largest fraction that is less than 1, by filling a cup up and pouring out drops or small amounts of water.

Figures 1 and 2. Students have a full glass of water represent one whole and break it apart to create fourths.

Creating a Number Line on the Cup

Connection to the Maryland College and Career Ready Curriculum Framework:

- *3.NF.A.2.A*
  Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
- *Knowledge of the meaning of the parts of a fraction (numerator and denominator)*
- *Knowledge of fraction 1/b as the unit fraction of the whole.*
- *Knowledge that when the denominator is 4, each space between the tick marks on a number line is ¼.*

Discussion questions:

- How are area models and linear models of fractions related?
- Why is it important to partition a shape or number line equally?
- How can you compose a fraction from unit fractions?
- How is partitioning water different than partitioning an area model using paper/pencil?
- What does each space between the tick marks on the number line represent?

Steps:

1. To teach this standard, students will connect a number line with the amount of liquid in their cup. Encourage students to benchmark a full cup as one whole and an empty cup as zero.
2. Using their learning from the previous activity, mark the given fractions on a number line with zero at the bottom of the cup and one whole at the top.
3. This is a good time to explain that if partitioning a rectangle to show a fraction (1/4), any one of the four parts can be shaded. However, when using a cup and water, since water is free flowing, the water at the bottom of the cup would represent the ¼. This is important as students begin to move from the concrete to visual models of showing their thinking and also makes a strong connection to linear and area models of fractions.
4. Using a blank cup, students will create a number line with markings for halves, thirds, fourths, sixths and eighths. Students may either compose unit fractions to make larger

fractions by pouring the water that represents a unit fraction into their cup repeatedly or create a number line and check their accuracy using water and cups.

Figure 3. Students label a cup with given fractions and connect these labels to number lines.

Comparing Fractions Using Water

Connection to Maryland College and Career Ready Curriculum Framework:

- *4.NF.A.2*
  Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
- *Ability to apply reasoning such as 5/12 < 1/2 because 6/12 is equivalent to one half so five twelfths is less than one half.*
- *Ability to identify the 'whole' for the fractions being compared.*

Discussion questions:

- Why is it important to use the same whole when comparing fractions?
- How can benchmark fractions help you compare fractions?
- What happens to the <, > or = when the order of the fractions you are comparing is changed?

Steps:

1. Using their cup from the previous activity as a tool, students compare fractions with different numerators or denominators. This can be differentiated by filling up two cups to represent two fractions and comparing the amount of water, or having students look at the number lines and decide which fraction is larger.
2. To teach the <, > and = symbols, create two fractions out of water and have students place an index card between them to compare them. When comparing to a benchmark fraction such as ½, it is beneficial to mark that on the cup using a different color,

highlighting the benchmark fraction that students are using to compare. In this case, the visual fraction model referenced in the standard is the cups and water, and students should practice drawing these (and connecting to a number line) as a way of showing their work. This is an appropriate time to lead a discussion about the importance of using the same whole (cup) when comparing fractions.

Figure 4. Students fill up 3/4 of the cup and 2/3 of the cup, and then compare the fractions.

Using manipulatives and other hands on measures in mathematics is not a panacea to improve mathematics instruction, but with comprehensive planning and reflection, it has been proven effective as one intervention to improve outcomes for students who are struggling, particularly with fractions (Laski, Jor'dan, Daoust, & Murray, 2015). Since the 1800s, various educators and researchers have advocated for the use of hands-on materials including: Jean Piaget, through concrete to representational to abstract teaching; Maria Montessori, through the use of structured materials with increasing levels of complexity; and George Cuisenaire with the invention of Cuisenaire rods, now found in thousands of schools worldwide (Furner & Worrell, 2017). I am hopeful that by utilizing something as simple and ubiquitous as water, teachers are able to channel these mathematics pioneers, creating an engaging and effective learning experience for children.

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Lessons Learned from Launching Tasks

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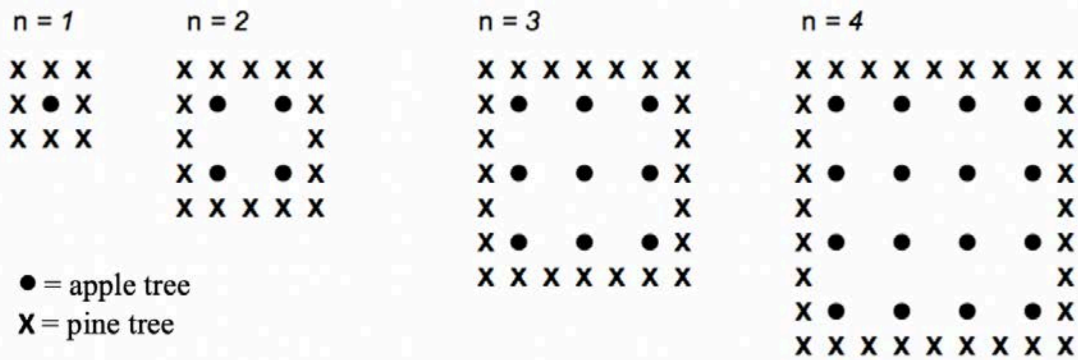
Abstract: *How a task is launched, or introduced, in the mathematics classroom has direct implications for which students can engage with the task and the kind of work the teacher engages with following the launch. In addition to sharing the criteria supporting an effective launch, we describe how two mathematics educators launched a task in the mathematics classroom. In considering how students were able to engage with the mathematics following the launch, we share reflections and lessons learned for others looking to improve how they launch tasks.*

It is well documented that classrooms should include tasks or activities that require students to consistently reason and make sense of mathematics (e.g., tasks that lack a clear solution path, require multiple representations, etc.,) (National Council of Teachers of Mathematics [NCTM], 2014; Stein, Grover, & Henningsen 1996). It is also documented that these tasks provide students with the greatest opportunities to understand mathematics but are the most difficult to implement well (Boston & Smith, 2009; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Recent work by Jackson and colleagues (2013) identified the launch, or how a task is introduced, as affecting key aspects of the task implementation, such as (1) which students can engage with the task and (2) the kind of work the teacher engages with following the launch (e.g., reexplaining task instructions to various groups versus supporting students as they struggle productively with the mathematics).

Jackson and colleagues (2013) also identified four criteria supporting an effective launch; each criteria is defined and discussed in the context of the Windbreak Task (see Figure 1). The task provides students an opportunity to identify and compare patterns between linear and quadratic relationships.

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants pine trees all around the orchard. Here you see a diagram of this situation where you can see the pattern of apple trees and pine trees for any number (n) of rows of apple trees:



a. Complete the table:

n	Number of apple trees	Number of pine trees
1	1	8
2	4	
3		
4		
5		

b. Describe the pattern (using words or symbols) so that you could find the number of **apple trees** for any stage in the pattern.

c. Describe the pattern (using words or symbols) so that you could find the number of **pine trees** for any stage in the pattern.

d. For what value(s) of *n* will the number of apple trees equal the number of pine trees? Show your method of calculating this.

e. Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of pine trees? Explain how you found your answer.

Figure 1. Windbreak Task adapted from PISA.

First, how *key mathematical ideas and relationships* are represented within the tasks should be explicitly discussed. For example, in the launch of the Windbreak Task, students might be asked: (1) What kinds of things do we look for when we're trying to identify patterns (increasing/decreasing, by how much, by addition/multiplication)? and (2) What kinds of mathematical symbols might we use when expressing patterns algebraically? Second, for tasks including a story or real-world context, the *contextual aspects* of the task should be discussed so students may engage with the task free of confusion regarding its surrounding context. Within the Windbreak Task, a discussion of how tress might be used as windbreaks, as wells as a picture of a fruit orchard with bordering trees, might be used to establish relevant context. Of note, for tasks that do not include real-world context, this aspect may be excluded from the launch (Jackson et al., 2013).

The remaining criteria—establish *common language and maintain cognitive demand—are* elements to consider while discussing the contextual and mathematical aspects of the task. In other words, contextual features and mathematical aspects are the medium in which the criteria of common language and cognitive demand are considered. One way to establish common language around key aspects of the task is to solicit feedback from a range of students to ensure all aspects of the task are well understood. With respect to the Windbreak Task, the previously listed questions around mathematical ideas and relationships would serve well in establishing common language as they require more than a *yes* or *no* response (Jackson et al., 2013). Likewise, these questions would need to be discussed across multiple students. Lastly, it is essential that the teacher not give away so much information that it lowers the level of reasoning the task requires, and thus, key learning opportunities (Stein et al., 1996). In the Windbreak Task, the teacher might avoid the terms "exponential growth" and "arithmetic growth" as to not cause cognitive interference for students in narrowing their focus as they reason and make-sense of the task.

In the following sections, we describe how two novice mathematics educators, Cindy and John, planned and launched a mathematics task in two different classrooms; both were completing student teaching requirements at the time of their task enactments and in approximately the eighth week of the school year. Although Cindy and John had read about the criteria of launching a task, and even engaged in a rehearsal around launching a task, we suspected much was to be learned from the planning and implementation of these elements within an actual secondary classroom. To help other mathematics educators improve in how they launch tasks, we close with lessons learned and suggestions for improving this practice.

Exponential Decay Task Launch

Cindy launched an exponential decay task found in the mathematics curriculum, *Connected Mathematics 3, Grade 8* (Lappan, Phillips, Fey, & Friel, 2014). This was the second lesson in the unit, which focused on exponentials and standards 8.F.A and 8.F.B. The problem description reads as follows:

> This problem focuses on the decreasing amount of active medicine in an animal's bloodstream in the hours following the initial dose. Question A gives a graph and a table for the decay pattern and asks students to find the decay factor and initial value. Question B presents the decay rate of another flea medicine by giving the decay rate (or percent decrease) of the medicine. Again, students identify the decay factor and initial value and write an equation for the population decay. (Lappan et al., 2014)

This task is cognitively demanding as it requires students to engage in non-algorithmic thinking around how to find the decay factor when they have not previously received explicit instruction and/or substantial opportunities to engage with this content (Stein, Smith, Henningsen, & Silber, 2000). Students were first asked to complete Part A of the task, engage in a whole-class discussion, and to then complete Part B of the task, again followed by a whole-class discussion.

Contextual Features

Cindy realized that not all students may understand how medicine is metabolized in living organisms and so she noted that the class would need some contextual background in this area. After having a student read the task scenario aloud, students were asked if they had ever taken medicine, and if so, how they were instructed to do so. Although most students had taken medicine, their understanding of why medicine is taken in repeated intervals was unclear. Because of this, the metabolization of medicine in both humans and animals was discussed briefly. As multiple students participated in this discussion, Cindy was confident that a common language around contextual features of the task had been established.

Mathematical Ideas and Relationships

In the previous lesson, the students were introduced to exponential decay; however, the rate of decay and decay factor were 50% and one-half, respectively. Cindy was concerned that students would struggle to calculate the decay factor from the rate of decay when the rate of decay was not 50%. To address this concern, Cindy conducted a brief review of exponential patterns and vocabulary from the previous day's lesson. More specifically, students were asked "Are exponential decay patterns also exponential functions? Why?" Students were quick to confirm that yes, an exponential decay pattern would be an exponential function; students' explanations were likely a result of the emphasis given to the mathematical structure of exponential functions in the previous lesson.

Common Language

Although common language was primarily developed within the discussion of contextual features and mathematical ideas, Cindy asked multiple students to *restate* in their own words what the task was asking them to do (Chapin & O'Connor, 2007).

Cognitive Demand

The cognitive demand of the task was maintained throughout the launch. As the key learning goal was around calculating the decay factor for a rate of decay other than 50%, Cindy focused her discussion on exponential patterns, relevant vocabulary, and the structure of an exponential function, all of which were mathematical ideas needed for students to productively engage with the task.

Reflection on Task Launch

Although students' efforts to reason and solve the task are not included within the launch, students' engagement with the task is often indicative of how well the task was launched. Because of this, we briefly discuss areas of success and improvement during the task implementation and how the launch may have affected each of these areas.

Following the launch for Part A, Cindy noted that all groups were able to begin working on the task. This is important as one indicator of an effective task launch is that students get to work quickly and are not reliant on the teacher to come and reexplain what they should be doing (Jackson et al., 2013). With respect to solving the task, many students were struggling to write the equation. Cindy asked students how they created the prior lessons' equations or asked them questions about writing equations for exponential functions in general. By the end of the task, various groups were able to create three different, but correct equations. Even so, Cindy hypothesized that an opportunity for students to discuss or practice creating an exponential equation would have been helpful, either at the start of class, or during the launch.

The implementation of Part B proved a greater challenge for students. With few exceptions, the students struggled with finding the decay factor to use in calculating table values—a key mathematical idea and relationship of the task. Likewise, Cindy reminded the class that the table values focus on not how much medicine leaves the blood, but how much medicine is *left active* in the blood—a key contextual feature of the task. Given the students' struggles, Cindy ended up leading students through the remainder of the task. Cindy felt a much more extensive launch should have been provided for Part B, even if it meant extending the task into a second day.

Centers of Triangles Task Launch

John launched a task examining the different centers of triangles in a high-school Geometry class; the associated content standard was G-CO.12. The printed task required students to take three different triangles, and using a ruler and protractor, find the incenter of one, the circumcenter of another, and centroid of the third. Following, students would conduct a series of investigations to identify commonalities, differences, and properties for each center. For example, students were asked to balance each triangle on its center. Does one of the centers balance the triangle perfectly? If so, why do you think this happens? Use mathematics to justify your reasoning. John's task is cognitively demanding as it provides students an opportunity to explore mathematical relationships and to then justify their reasoning (Stein et al., 2000). Students were asked to complete the task in groups and to be prepared to engage in a whole-class discussion. Students had been studying triangle congruence and centers of triangles; relevant mathematical language had also been established (e.g., perpendicular bisectors, angle bisectors, segment bisectors). Note that since the task lacks contextual features, this discussion is not needed (Jackson et al., 2013).

Mathematical ideas and relationships

Students had previously defined the three centers of a triangle, but not been provided opportunities to practice finding each of the centers, or explore the similarities, differences, and additional properties of and across the centers. Because of this, John reviewed the definition of each center of a triangle and associated vocabulary (e.g., perpendicular bisectors, angle bisectors, medians). At the time of the launch, John was satisfied that a common language specific to mathematical ideas and relationships had been established through questioning, many of which required more than a yes or no response (Jackson et al., 2013).

Common language

Beyond developing a common language through discussion of mathematical ideas, John spoke with the class around what the task entailed and provided an overview of specific instructions. As student responses were not required, there is limited evidence that John established common language around task expectations and instructions.

Cognitive Demand

The cognitive demand of the task was maintained throughout the launch. As the key learning goals were to practice finding each center, and to then explore the properties of those centers, John focused his discussion on the definitions of each center, as well as relevant vocabulary, all of which were mathematical ideas needed for students to productively engage with the task.

Reflection on Task Launch

Following the launch, John reported that the class did not understand the details of the task. Several students asked John what they were supposed to do with the cut-out triangles, which centers to draw and how to draw them, while others would complete one triangle and then wait to be told to begin on the next triangle. Although John himself discussed the directions and details of the task, he did not ask any of the students to restate what the task was instructing them to do. John felt he should have spent more time establishing a common language around the relevant vocabulary and skills needed to construct the triangle centers (e.g., perpendicular bisectors, angle bisectors, medians, etc.). It is possible students could have practiced and/or applied each of these skills at the start of class. John also suggested chunking the task into smaller portions, each with their own targeted task launch, perhaps by each triangle center.

Further, John did not anticipate the difficulty students would have taking measurements with a ruler. Subsequently, students were confused in using their calculators to divide each triangles' measurements. Several of the students used inches when measuring the sides of the triangle and did not recall that the inch was divided into sixteenths; they assumed it was divided into tenths. Upon noticing this problem, John asked students to use the metric side of the ruler. John realized he should have specified within the launch which measurement system to use; he also felt an opportunity for students to measure and identify the midpoint of an item prior to the task would have been helpful. Although John did not plan for the task to span two days, given the students' difficulties in completing the task, the task was continued a second day. During the second day of class, a number of students were able to find their triangles' centers and were able to begin reasoning about various attributes specific to each center.

Lastly, John noted that an additional detractor from the task was student behavior. The classroom culture was very relaxed, with many of the students listening to music and having side-conversations. Although not directly related to mathematics, John noted that he should have reminded students of behavioral expectations during the launch and implementation of the task.

Lessons Learned

Following opportunities for both Cindy and John to launch tasks, suggestions for other mathematics educators as they consider planning and launching similar tasks were considered.

- **Overemphasize the establishment of common language.** In both cases, how well Cindy or John did or did not develop common language had an impact on how students were able to engage with the task. With such implications, we recommend being intentional in overemphasizing the development of common language with respect to both contextual features—when relevant—and mathematical ideas and relationship. One-way Cindy was able to effectively do this was to have multiple students restate the task instructions (Chapin & O'Conner, 2007), as well as ask questions that move beyond a yes or no response (Jackson et al., 2013). In contrast, although John felt the mathematical ideas and task instructions were fairly straight-forward, because his efforts to establish common language were minimal, students' ability to engage with the task was limited. *Regardless of how well the teacher states the contextual features or mathematical relationship within the launch, if these ideas are not shared by the students, the launch and subsequent task implementation will not be effective.*

- **Use a task launch planning framework.** Both Cindy and John made use of a task launch planning framework developed by Jackson and colleagues (2012), which includes a set of planning questions around each of the four criteria and mathematical goals of the lesson. For example, for contextual features, teachers might ask themselves "*Which features are likely to be unfamiliar to some or all of my students*?" (p. 29).
- **Thoroughly consider and discuss the mathematical aspects of the task.** A difficult balance exists between discussing an appropriate amount of mathematics and not lowering the cognitive demand of the task. We found focusing the launch on the mathematics students would need to reason within the task—prerequisite skills—to be most productive. The previously mentioned planning framework (Jackson et al., 2012) also provides specific prompts that might help teachers consider which mathematics is most appropriate to discuss, while not lowering the cognitive demand. For example, teachers might consider **"***What key mathematical ideas do my students need to understand so that they will be able to engage in solving the task*?" (p. 29).
- **Chunk the task into manageable parts.** If different mathematical relationships are needed at various stages of the task, break the task into chunks that allow relevant and important mathematics to be discussed intermittently throughout the task (e.g., multiple launches within the same task). For example, John might have chunked the task by having students focus on locating one center at a time. Following, launch the latter portion of the task that asks students to explore properties, similarities, and differences across the centers.
- **Plan for more time than you think will be needed.** Time can be a valuable commodity in a classroom, but there needs to be enough time allotted for students to fully engage with all criteria of the task. For example, because Cindy had not planned for the task to span beyond a single day of instruction, Part B of the task was not properly launched and the cognitive demand of the task was eventually lowered as Cindy had to complete the task whole-group. If teaching for understanding, opportunities, and thus time, are needed for students to reason and make sense of the mathematics.
- **Be kind to yourself.** Learning to effectively launch a task is a teaching practice that will likely take practice and some experience. Remain reflective on your practice around each of the criteria of an effective task launch, just as John and Cindy have.

Conclusion

Task-based learning is more difficult for teachers to implement than direct instruction, but the *learning*–not memorization–that students experience make it a most worthwhile endeavor (Skemp, 1978). To ensure students have appropriate opportunities to make sense of and reason with challenging tasks, the launch must provide the appropriate information, all while maintaining the cognitive demand of the task. In addition to establishing a common language through the discussion of contextual features and mathematical ideas and relationships, we have provided a set of helpful tips when launching a challenging task.

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Going Beyond the Standards: Implementing Research-Based Activities to Introduce Fractions in Second-Grade

Kathryn Trembley, McDaniel College
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Abstract: *The Common Core State Standards introduce fractions through partitioning shapes in the geometry standards in first grade. However, recent research suggests that building on students' intuition of fair-sharing and division might help develop a more conceptual understanding of fractions as numbers. This article describes two activities utilizing fair sharing created and implemented by a pre-service teacher in a second-grade small group lesson. She reflects on her experiences using research to inform her planning and evaluate the outcomes of her lesson.*

Although the Common Core introduces students to the idea of fractions through area models in the geometry standard in first grade, research suggests that partitioning shapes may not be the most effective way to build students' conceptual understanding of fractions as numbers.

> 1.GA.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. (Common Core State Standards Initiative, 2010)

One recent publication has shown that students as early as kindergarten can understand early fraction concepts when presented through partitioning and fair-sharing activities (Cwilka, 2014). Other research (Empson & Levi, 2011) indicates that the area model may not be the best way to introduce fraction concepts to new learners and that building on children's early thinking about multiplication and division, specifically dividing into equal groups (*sharing*), provides the foundation for learning and understanding fractions as numbers. Additionally, McNamara and Shaughnessy (2010) demonstrate the power of exposing students to multiple representations of fraction concepts, as well as the importance of considering the work of classmates to further develop students' conceptual understandings of fractions.

When teaching focuses primarily on procedures disconnected from developing conceptual understanding through relevant contexts, we can inadvertently foster misconceptions in our students (NCTM, 2014). One misconception that develops frequently is that students see the numerator and denominator as separate entities and do not understand that the fraction is a number that represents the *relationship* between the numerator and the denominator (Siegler et al., 2010). Fair-sharing contexts help to develop an understanding of that relationship between the divisor and the dividend or the numerator and denominator when the division is represented by a fraction (Cwilka, 2014).

This article describes two activities designed to help second graders develop a conceptual understanding of fractions through partitioning and fair-sharing activities that was implemented by a pre-service teacher in a practicum placement prior to student teaching. The authors want to promote the idea that teachers can build on children's developmental and intuitive understanding of fair-sharing *before* these ideas are formally introduced in the standards. The article also includes one author's (Ms. Trembley's) reflections on implementing these research-based activities in her small group instruction to provide the personal insights from a beginning teacher's perspective on going beyond curricular materials and grade-level standards to realize current research suggestions.

Ms. Trembley decided to implement activities utilizing both fair sharing and partitioning to assess the effectiveness of this approach for building conceptual understanding about fractions. Using the method described in Wilson, Myers, Edgington, and Confrey (2012), she developed a scenario that required students to first fair-share a collection of items before moving on to the dividing and sharing of a whole. The ordering of these activities is important because it helps to build on children's experiences of sharing sets of objects (e.g., blocks, crayons, cars, etc.) before partitioning a single unit. Since it was early November, she had the students think about sharing Halloween candy in this enhancement activity for six high-achieving second grade students (four boys and two girls). The activities described below are an example of a starting point in developing conceptual understanding about fractions. These activities use "friendlier" numbers, but the authors would recommend that subsequent activities utilize a variety of numbers, including odd numbers, and make explicit connections between the different numbers to prevent fostering misconceptions around the "types" of numbers that can create fractions.

The Great Halloween Heist

Activity 1: Partitioning a Collection into Shares

The context of this activity was adapted from pirates and buried treasure (as in Wilson et al., 2012) to friends trick-or-treating, as the small group lesson fell directly after Halloween. This change was made to make the task interesting and relevant to the six students participating in the fraction lesson.

> The problem: *This year for Halloween, you and some friends had a brilliant idea. Rather than haul your own bag around and have your arms get tired, you decided to combine your Halloween candy in a wagon that you would pull while trick-or-treating. At the end of the night, you bring the all of the candy home and split it between you and your friends. The question is, how are you going to divide this candy up equally?* This cup of 30 colored

counters is all of your candy. With a partner, divide this up equally if two people went trick-or-treating together.

Working as partners, the students shared the "candy" (modeled with counters during the lesson) among an increasingly larger group of friends and charted their findings together. They used the following criteria for fair sharing: (1) each friend present must get a share (creating the correct number of groups or parts), (2) each share must be equal, and (3) there cannot be any leftover candy; the whole bag must be shared.

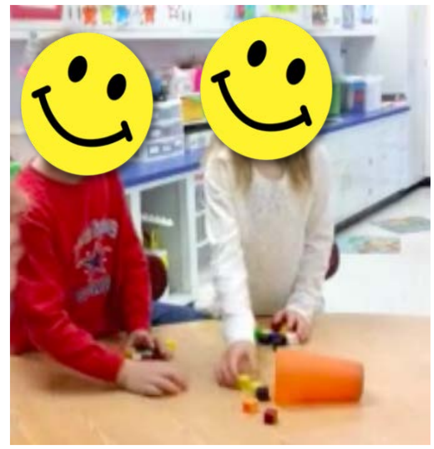

Fig. 1. Two second-graders use counters to model a fair-sharing problem.

Each pair received a container with 30 counters in it. Ms. Trembley asked, "How could you fairly share the candy among friends?" Students then explored sharing the thirty counters among 2, 3, 5, 6 and 10 friends successively. As they worked, the students were asked to share their strategies for dividing up the whole, as well as show how they knew each share was equal. Ms. Trembley used questions to elicit student thinking about important fraction concepts:

- Strategies for fair sharing:
  - How did you share equally among the friends?
  - How do you know your shares are equal?
- Understanding equal-shares: Which share of the candy would you rather have?
- Exhausting the whole: What about the leftover pieces?

Students explained their process of partitioning the candy stash into each group and discussed how they knew they were equal as a way to make their thinking accessible to other students. Ms. Trembley recorded the number of objects per group, and then helped the students to name the different shares, such as *one-half*, *one-third*, *one-fifth*, etc. By the end of this activity, the completed chart looked like the following:

Whole candy stash: 30 pieces		
Number of Friends	Number of Pieces of Candy	Name of the Share
2	15	*one-half* 1/2
3	10	*one-third* 1/3
5	6	*one-fifth* 1/5
6	5	*one-sixth* 1/6
10	3	*one-tenth* 1/10

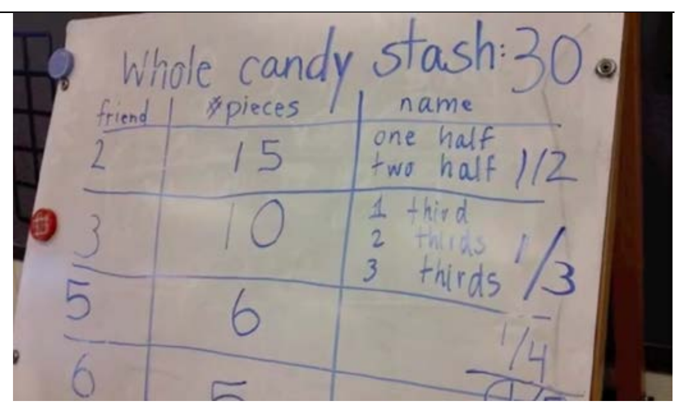

Fig. 2. White board recording of Ms. Trembley's work with the second-graders.

After the table was full, students were encouraged to make connections between the number of friends, the size of the share, and the name of the share:

- Which group would you like to go trick-or-treating with and why?
- What patterns do you notice with the number of candy pieces when the bag is shared with more friends?
- Look at the way we named the shares. What is happening with the name of the shares?

A connection between the number of friends and the denominator was established through questioning before moving on to partitioning one whole: "Why is the denominator the same number as the number of friends?"

Activity 2: Partitioning a Whole into Parts

The next scenario was that, for some reason, the whole stash of candy melted into one whole candy bar. The candy still needed to be partitioned across friends, with each friend receiving an equal sized piece and no candy remaining. The students received square pieces of paper and were asked, "How can you help the friends fairly share their candy bar?" As before, they first shared between two friends. Students determined how best to partition the whole, given scissors, rulers, and pencils. When they completed the task, they were asked to attend to the criteria and explain their reasoning: "How do you know the pieces are equal?" or "Which piece of the candy bar would you want? Do you think the candy bar is shared equally?" to re-establish equal sharing.

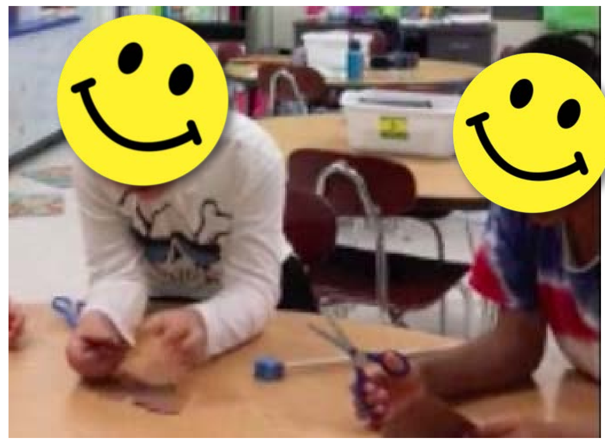

Fig. 3. Two second-graders use paper and scissors to model another fair-sharing problem.

After sharing between two friends and allowing students to share their reasoning for partitioning and equality, students were asked to name each of the pieces: "If this was one whole candy bar, what could I name this one part of the candy bar?" The pieces were labeled and glued to chart paper. The students were then given another piece of paper and asked to share the candy bar equally among four friends. Again, they used the tools of their choice and determined how to show the pieces are equal. In a similar way to the halves, they named the pieces and wrote the name on the piece, before gluing it to the chart. This process was repeated for three friends and five friends. Again, Ms. Trembley emphasized the size of the share against the number of partitions, tracked student answers on a chart and encouraged them to look for patterns in the numbers.

When the pieces were labeled and glued, both the chart and the white board findings were placed in front of the group. The organization of this recording was important to help students make connections between the mathematical ideas. Ms. Trembley was purposeful about how she represented the fractions in both the area model and the numerical representation. Formally naming the denominator, discussing its meaning and extending to other partitions was the wrap-up of these two activities. The questions she used to once again elicit student thinking were:

- What patterns do you notice between the names of the pieces compared to the whole bag of candy or the whole candy bar? Looking at the charts, what do you think the denominator tells us?
- What if I shared among eight friends? What fraction could I write to name each share?

The two tasks outlined above involve introductory inquiry-based activities to develop to an understanding of what the denominator means in fraction notation building on students' developmental experiences of sharing objects. Manipulatives were an important part of this activity, allowing students to physically partition pieces into different groups and draw/cut a whole into equal parts. Finally, the shares are named using mathematical notation and the denominator discussed *after* students are encouraged to observe patterns. For the next activity, Ms. Trembley would develop an understanding of the numerator, and how the two parts of a fraction tell us different things about the pieces and the whole and begin to introduce the length/linear model (e.g., a number line) for students to further develop their understanding of fractions as a number. Again, students need a variety of experiences and activities that begin with building on their own intuitions and experiences, but then push them beyond those using all three models to fully develop flexible understandings of fractions.

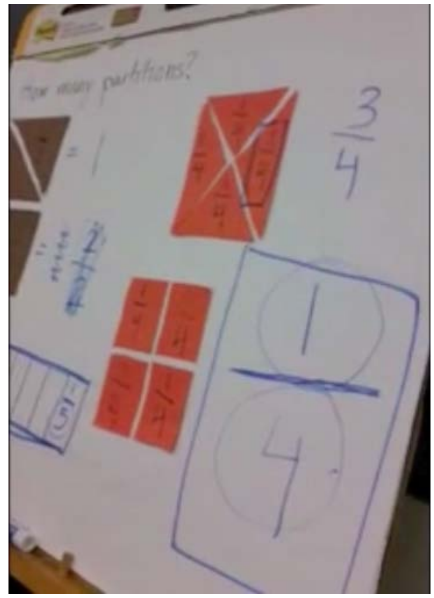

Fig. 4. Recording of area model partitions.

Ms. Trembley's Reflection on the Activity

The lesson I taught was an enhancement activity for six high-achieving 2nd graders. As I pulled the students from their classroom and walked with them down the hall and to our workroom, all six students were pumping their fists and exclaiming, "Yeah! Fractions!" Given the dislike and resistance of fraction learning with both upper elementary students and adults, this reaction surprised me. I was excited to implement the research with my group to see what they did with the activities, and they were excited to explore a topic normally withheld from them until third grade.

As I considered how to apply the research of partitioning and fair-sharing to an introductory lesson on fractions, I wanted the activities to be student led. I wanted students to come to fraction and part/whole understanding through their own exploration, as well as to see what connections they would make to previously learned concepts. Our discussions were meant to tease out patterns and stretch understanding to deeper levels, as I introduced formal terminology to support the ideas they developed during the activity. Two questions drove our discussion and evaluation of student thinking: How are you going to partition the collection/whole? How do you know the shares/pieces are equal?

The first question focused on strategies to partition the whole, such as dealing, dividing, counting by two and other strategies listed in Wilson et al. (2012). From this activity, I could gauge sophistication in problem solving, as well as the connections made to other mathematical concepts for each group of students. When dividing the collection at first, most students implemented a dealing method, separating into groups by one, two or even five. However, one group of students used their knowledge of division to quickly count up the pieces (by fives) and divide the whole set into two groups. As we moved through the lesson, more students began implementing similar strategies of increased sophistication to match the demand of the problem. When separating the collection among five friends, students used knowledge of basic multiplication facts to quickly form their groups, while others skip counted by five. At this point, I could see most students' thinking had moved beyond the manipulatives and into more abstract thinking.

Conclusion

Through this activity, students explored fraction concepts first through a discrete model (counters), followed by an area model, instead of only through the area model as suggested by the Common Core. Starting with "halves," students were encouraged to name each share of the collection and the whole, linking fraction notation with the written labels of halves, thirds and fourths, as encouraged in Wilson et al. (2012). Displaying student thinking allowed each student to consider the process of their classmates and begin to understand flexibility in partitioning a whole, as well as address misconceptions about fraction concepts, which also aided in developing of students' conceptual understandings of fractions (McNamara & Shaughnessy, 2010). By charting results in the discrete model and creating a display of student work when partitioning the whole, each student had access to the thinking of their classmates and was able to make connections between the mathematical ideas.

The experiences of a pre-service teacher's implementation of research on developing fraction understanding help demonstrate the positive learning outcomes that accompany such effort. Students were able to successfully meet many of the Standards for Mathematical Practice (CCSSI, 2010) in the activities above, such as SMP 1 Making sense of problems and persevering in solving them, SMP 2 Reasoning quantitatively and abstractly, SMP 3 Constructing viable arguments and critiquing the reasoning of others, and SMP 7 Look for and make use of structure. They also helped to develop a solid understanding of the foundations of rational numbers by seeing fractions as numbers that describe a relationship, and not solely as parts of a shape. Although, the Common Core State Standards (CCSSI, 2010) have provided a solid framework for the elementary curriculum, especially when it comes to fractions, teachers might want to consider seeking out peer-reviewed or published activities and adding them to their own instruction. In order to provide the most effective learning experiences for students, the authors encourage all teachers to develop lessons that go beyond the standards.

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Kathryn Trembley studied elementary education at McDaniel College in Westminster, Maryland. She is currently working toward a Master of Science in Elementary Education focusing on education equity and social justice in K-3 mathematics. As a Teach for America candidate, she taught 3rd grade in Sunnyside, Washington while completing an accelerated teacher certification program through the University of Washington. Kathryn is interested in implementing discussion and inquiry-based mathematics to promote deep understanding of mathematics concepts and attempts to improve her teaching practice by considering current educational research in all aspects of her classroom.

Barbara Swartz has professional interests in preparing successful teachers of mathematics at all levels with a passion for designing learning opportunities to help learners make sense of the mathematics. She teaches mathematics courses, pedagogy courses for both elementary and secondary mathematics, and technology integration courses at McDaniel College. She earned her doctorate from University of Virginia.

Job-Embedded Professional Learning in Two Schools

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Denise Bogart, Howard County Public School System

Abstract: *In this article, two elementary mathematics coaches describe their experience with job-embedded professional learning in their respective schools. They review models they have designed based on attributes of effective professional learning, provide examples for each model that mathematics coaches can easily replicate, and share pre-planning and debriefing forms they have used with teachers. They also present data collected in a survey about job-embedded professional learning. Finally, they share their administrators' perspectives and their own experiences with implementation. It is their hope that through reading this article, mathematics coaches and teachers might be compelled to try this professional learning model.*

A Distinct Process for Professional Learning

The work of a mathematics coach is grounded in establishing trust with teachers while supporting their teaching, to improve student learning.

> "How do you bring teachers' attention to math content, to the significant aspects of student thinking, and to the impact of their actions in the classroom? How do you help teachers develop skills and habits of preparing for a lesson, of eliciting student thinking, and of analyzing student work?" (Schifter, 2009, p. viii).

These critical questions have influenced our work as mathematics coaches, compelling us to probe effective models of professional learning. Guiding our inquiry was a report from the Learning Policy Institute titled *Effective Teacher Professional Development*, which cited seven important elements of professional learning: is content focused; incorporates active learning: supports collaboration typically in job-embedded contexts; uses models and modeling of effective practice; provides coaching and expert support; offers feedback and reflection; and is of sustained duration (Darling-Hammond et al., 2017, p.23). You will see these principles illustrated in the models we discuss in Table 1.

The purpose of this article is to describe our experience with job-embedded professional learning in each of our respective schools, both Title I, in Howard County, Maryland. Title I is a federally funded program "…that provides financial assistance to local educational agencies (LEAs) and schools with high numbers or high percentages of children from low-income families to help ensure that all children meet challenging state academic standards" (U.S Department of Education, 2018). We will do this by reviewing models we have developed based on attributes of effective professional learning, providing examples for each model that mathematics coaches can easily replicate, and sharing pre-planning and debriefing forms we have used with teachers. We will also present data collected in a survey about job-embedded professional learning. Finally, we will share our administrators' perspectives and our own experiences with implementation.

Optimizing Differentiation through Job-Embedded Professional Learning

Job-embedded professional learning has given us the ability to differentiate support for teachers. Like students, each and every teacher we collaborate with is at a different level of both content and pedagogical knowledge. To foster professional learning that is relevant, builds in feedback, and supports teachers as they apply new learning in the classroom, we have designed and adjusted particular ways to implement job-embedded professional learning in our schools (Zepeda, 2012, p.11). When implementing these models, keep in mind that the choice of model may be flexible and adaptable. These models may be used interchangeably depending on the teacher or grade-level team needs, content knowledge, and student data.

Table 1 provides descriptions and examples of job-embedded professional learning models we have utilized.

Table 1. *Job-Embedded Professional Learning Models*

Model	**Attributes of Effective Professional Learning**	**An Example**
Individualized planning, co-teaching based on teacher's needs, and debrief	• Relevant to the individual teacher • Facilitates the transfer of content or skills into practice • Duration of support is sustained from three days to one week • Feedback is built into the process	*Mathematics coach collaboratively plans with a first-year teacher for three days of lessons focused on a content topic such as the introduction of fractions or models a skill such as utilizing focusing questions. Mathematics coach co-teaches with the teacher, and incorporates debriefing after each day of co-teaching, with a final reflection meeting at the end of co-teaching.*

Team collaborative planning, co-teaching based on teachers' needs, and debrief	• Allows teams to share and consider different perspectives • Supports teachers in trying new strategies and learning together • Facilitates the transfer of content or skills into practice • Feedback is built into the process • Duration of support is sustained and can be on-going throughout the year	*Mathematics coach collaboratively plans with a grade level team on a weekly basis, schedules a two to three-day co-teaching experience with each teacher on the team (a teacher can request a content topic or work on a pedagogical issue such as facilitating productive struggle), incorporates debriefing after each day of co-teaching with a final reflection meeting at the end of co-teaching.*
Team professional learning, planning and co-teaching with individual teachers, and debriefing of the lesson based on student work	• Professional learning is content focused • Creates an openness to learning from data • Facilitates the transfer of content or skills into practice • Feedback is built into the process • Duration of support is sustained and can occur for an entire month.	*Mathematics coach facilitates professional learning on a topic such as building procedural fluency from conceptual understanding with addition and subtraction, schedules two to three days of co-teaching with each teacher on the team (planning of lessons can be differentiated based on the particular needs of students in the class), incorporates debriefing after each day of co-teaching, with a final team data meeting at the end focused on student work.*
Team professional learning, planning and modeling for individual teachers and debrief	• Supports teachers in trying new things and learning together • Incorporates modeling • Duration of support can be up to two weeks for the entire team.	*Mathematics coach leads professional learning on a topic such as "Using Breakout Boxes in the Mathematics Classroom," schedules one day of modeling for individual teachers in a team, and debriefs after the modeling experience, schedules co-teaching with teachers with a goal of teachers developing confidence in utilizing Breakout Boxes in their own classroom.*

In our work with teachers, pre-planning and debriefing were vital components of job-embedded professional learning. While these took time to implement, we cannot overemphasize the value gained from clarifying for both the teacher and mathematics coach what the focus would be for planning content or the mathematics teaching practice, co-teaching, and student data collection. A pre-planning form such as the one provided in Figure 1 is completed by the teacher and then reviewed by the mathematics coach.

Job-Embedded Co-Teaching Planner

CONTENT: Check the content area you would like for us to focus on for co-teaching:

Procedural Fluency

- ☐ Addition
- ☐ Subtraction
- ☐ Multiplication
- ☐ Division
- ☐ Fractions

Conceptual Understanding

- ☐ Addition
- ☐ Subtraction
- ☐ Multiplication
- ☐ Division
- ☐ Fractions

Use this space to say more about the area you chose as a focus:

____
____
____
____
____
____

PRACTICE: Check a practice you would like for us to focus on for co-teaching?

- ☐ Making sense of problems
- ☐ Reasoning using precise vocabulary
- ☐ Connecting representations
- ☐ Building Stamina
- ☐ Implementing the 5 Practices for Orchestrating Productive Mathematical Discussions
- ☐ Implementing the structures of a math class: number routines, rich task, closure
- ☐ Assessing with feedback
- ☐ Engaging students in math class
- ☐ Ensuring access for students with unfinished learning
- ☐ Other

Use this space to say more about why you would like to focus on this practice:

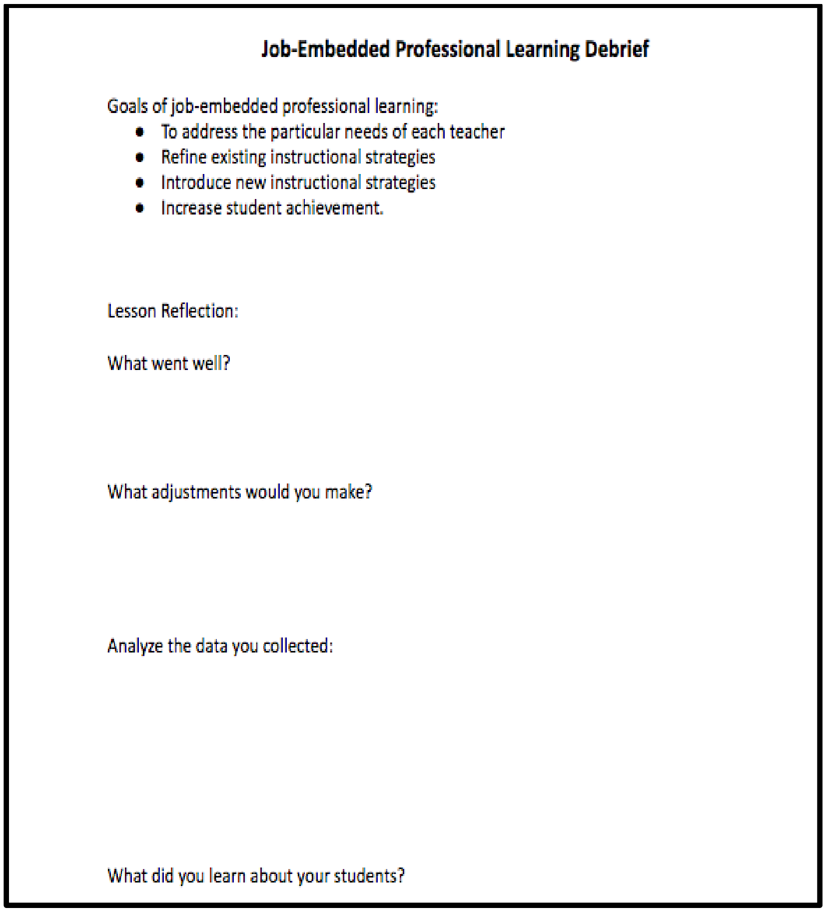
____
____
____
____
____
____

What else can I do to support you?

____
____
____
____
____
____

Figure 1. Pre-Planning Form

The debriefing form shown in Figure 2 guided both the mathematics coach and the teacher in planning where to go next. During this time the coach could identify the type of future support needed for the teacher and determine the effectiveness of student learning. In addition, the teacher was able to reflect and make adjustments to their teaching. This conversation between the coach and teacher was crucial for deciding next steps in the job-embedded professional learning process.



Job-Embedded Professional Learning Debrief

Goals of job-embedded professional learning:

- To address the particular needs of each teacher
- Refine existing instructional strategies
- Introduce new instructional strategies
- Increase student achievement.

Lesson Reflection:

What went well?

What adjustments would you make?

Analyze the data you collected:

What did you learn about your students?

Figure 2. Debriefing Form

Teachers' Perspectives

To refine the job-embedded professional learning models we implemented in our respective schools, and as a way to inform our School Improvement Plans (SIP) for the following school year, we collaborated on a survey for our staff to complete.   First, we asked, "Which of the following statements best describes how this type of professional learning compared with other professional learning in which you have participated during the past six months?

Figure 3.
School A

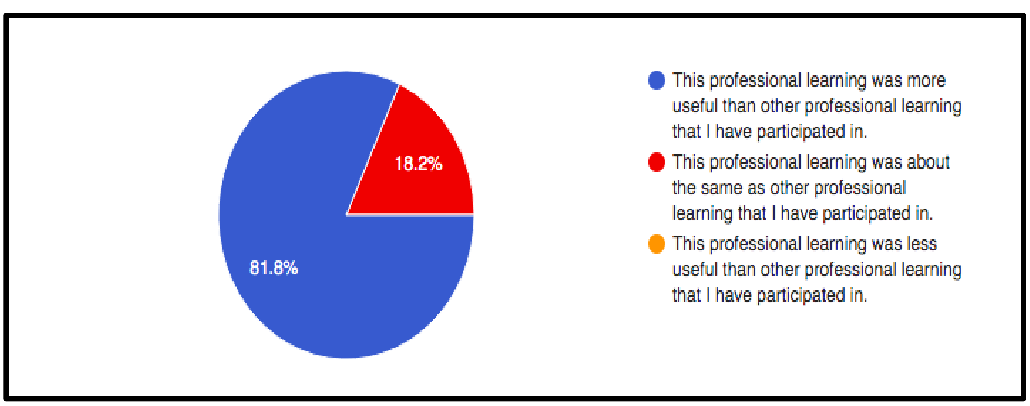
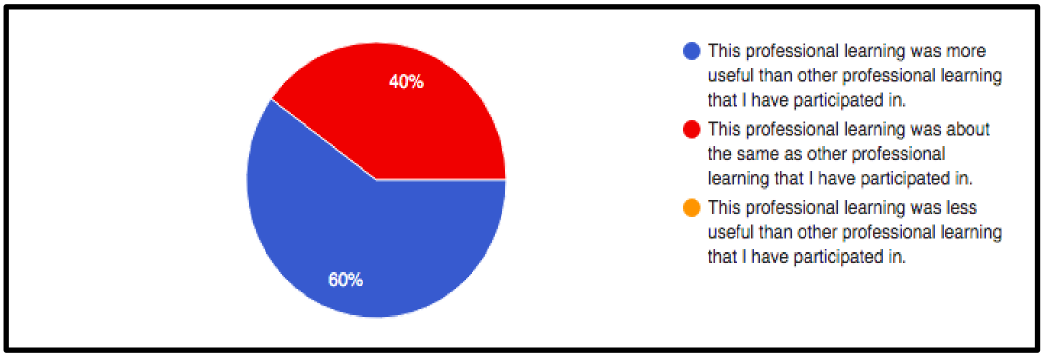


Figure 4.
School B



Note that this was the first year of implementation of job-embedded professional learning in School A, while School B was in its third year of implementation. What we have found promising is that in both schools, the majority of teachers still considered this form of professional learning as more useful than other professional learning experiences they have encountered within the given school year. We then asked, "Did your experience planning, teaching, and reflecting on a lesson or lessons with the mathematics coach improve your mathematical knowledge (building conceptual or procedural knowledge) or practice (referring to the mathematics teaching practices)?"

Around 64% of teachers in School A selected "practice" while 36% of teachers identified "knowledge." In School B, 80% percent of teachers selected "knowledge" while 20% percent of teachers identified practice. These variations in teacher responses were dependent upon what the individual teachers or grade level teams at both schools selected as their focus for job-embedded professional learning or the mathematics coach's topic for professional learning which is aligned to each school's SIP. Finally, teachers in both schools were asked, "How have you applied this new or refined knowledge or practice in your mathematics class?"

These are samples of the responses that we received:

> Respondent 1: "One way I have refined my practice has been by selectively calling upon students to share their thinking in an order that further develops conceptual understanding. Having students with concrete examples share first, followed by pictorial representations, followed by those that have abstract reasonings."

> Respondent 2: "My math coach modeled two different number routines in my classroom- *Just the Facts* and *Fraction Splat*. In using *Just the Facts* I learned questioning techniques to help students make the connections between the facts...I had never done *Splat* with fractions and was nervous to try it. I felt more confident after observing this routine and now use it frequently. "

> Respondent 3: "Having time to plan together was a great help to me this year. As I was new to teaching our enriched Kindergarten class, it was very beneficial to me to have time together regularly just so we could check in and for me to ask questions that came up as I taught new concepts. Teaching equality with a number balance was new to me this year.

> After a demonstration from my math coach and time to explore, I felt confident using the number balance with my students and it gave them the opportunity to deepen their understanding of equality, addition, and subtraction in a meaningful way."
>
> Respondent 4: "Both embedded PLs and regular sit-down PLs have been very useful for me, but the collaboration of the embedded PLs has been very rewarding!"

Administrators' Perspectives

In *Job-Embedded Professional Learning Essential to Improve Teaching and Learning in Early Education*, the authors emphasize, "to build trusting relationships for learning, job-embedded professional learning must be strengths-based, with an emphasis on becoming ever better, not focused on deficits. This creates an openness to learning from data versus defensiveness and allows teachers to respectfully share and consider different perspectives. It also supports teachers in trying new things, risking failure, and learning from all of it together" (Pacchiano et al., 2016). These conditions for teacher learning were reflected in our deliberations with administrators, and their observations related to these principles are further discussed below:

School A: After integrating job-embedded professional learning for one year at School A the principal described it as a "hands-on professional learning with the students who teachers work with each and every day. It makes the professional learning REAL." The assistant principal stated, "Our math coach was able to quickly build relationships with teachers and students as well as learn the culture of our school by being in the classrooms." The administrators also felt that in this professional learning model, "teachers are more likely to implement strategies that they have seen work successfully with their students in their classroom. Additionally, our math coach was able to help teachers identify challenges and find solutions to increase learning for both teachers and students."

School B: School B has implemented job-embedded professional learning for three years; each year the model was adjusted based on staff and student needs. Most importantly, the principal expressed that this professional learning model provided opportunities for "teachers to begin to realize that their learning should mirror the way they want their students to learn."

Administration also noticed that teachers who have used this professional learning model for the past three years have shown growth professionally, were more apt to accept constructive feedback, and would reach out to the mathematics coach on their own for support. Furthermore, the principal noted that based on her conferencing with teachers as well as academic walkthroughs and observation data, she has seen tremendous growth in the teachers' craft. The principal also explained, "It is essential for your school to build job-embedded professional learning as part of the culture of the school; you want the teachers to understand that teaching with the coach is part of learning. Job- embedded professional learning uplifts the value of continuous learning in our school building. It aligns with our school culture where we 'check our titles at the door' and do what is best for our students."

Two Coaches' Perspectives

In this section of the article, we, the authors, discuss our individual views about job-embedded professional learning and recount our experiences with this unique process.

Coach 1, School A: For the 2018-19 school year, I was assigned to a new school. My priority was to build relationships with teachers while focusing on my school's SIP for mathematics—building procedural fluency through conceptual understanding, particularly in the Numbers in Base Ten and Fractions domain. The reading coach who was also newly reassigned to my school suggested we use the job-embedded professional learning model she had implemented at her old school. She explained that it would go through the cycle of: professional learning → planning → co-teaching → debrief.

I began by co-teaching with a kindergarten teacher in her second year of teaching. She wanted to work on implementing the structures of a mathematics class, particularly implementing number routines and closure. We planned three days of lessons for the following week and then met during her planning time after the first day that we co-taught. During this meeting, she wondered whether she should have used a ten-frame with students, or if she should have focused on using a rekenrek. This was an opportunity for me to share that she could actually use both representations, in addition to a number path, as long as she was intentional about connecting the representations. She was encouraged by this feedback and generated the idea of highlighting a number path for representations of quantity.

The following week, a fourth-grade teacher well into her twentieth year of teaching and who was about to start her instruction on fractions, shared with me her hesitation about using Cuisenaire rods (C-rods). She loved using fraction towers, and fraction circles, but she was not as comfortable with C-rods. I suggested that we could co-teach together and use lessons from Learning Mathematics through Representations (from the University of California, Berkeley). The following is what she tweeted, "Thank you for working with my students today. We learned so much about fractions, including unit intervals and subunits!"

To me, the significance of job-embedded professional learning is that this model challenged me as a mathematics coach to be more reliable and accountable to teachers, which are important elements to building trust (Brown, 2018). I was able to teach with each teacher in each grade for three consecutive days, learning about their students and the dynamics of their particular classroom. Teachers were able to see that I too may have struggled or questioned the strategies or models I chose for a given lesson, and by bringing this to their attention, there was reciprocity in how we problem solved together about adjustments we needed to make. The process of professional learning → planning → co-teaching → debrief also allowed for more side by side coaching and I was not perceived by teachers as authoritative. In addition, learning happens for both the teacher and coach. For example, while co-teaching with a first-grade teacher who asked me to focus on addition on a number line, I noticed that students were still jumping by ones on the number line provided. When I tried to model jumps of ten for them, they each looked at me in confusion. In reviewing our county's scope and sequence of instruction, I realized that mentally adding 10 more or 10 less was not yet taught. I would not have caught this misalignment if I did not have the opportunity to go through the same process as the teacher I was supporting.

Coach 2, School B: I have utilized job-embedded professional learning for the past three years as a mathematics coach. Each year, I have adjusted the design based on the needs of the school, teachers, and students. It was essential for me to build trust and relationships first. I spent my first year working directly with teachers. Often, I had to remind them that we had the same goal, which was to support student academic needs. I also worked on ensuring this model of professional learning became part of the school culture and I had to emphasize that it was not a way to "fix teachers."

The next two years implementing the job-embedded professional learning model worked more efficiently now that I had built relationships and trust with teachers. I was now able to execute more of the true attributes associated with job-embedded professional learning such as planning with individual teachers or grade-level teams around their specific content needs, co-teaching or modeling lessons, providing regular feedback, and reflecting together to decide next steps based on student data. The pictures below are an example of a lesson I taught with each second-grade teacher. I taught with each teacher for five days working on concepts related to the clock, making the connection to partitioning shapes, and telling time. The students and teacher were able to see how this lesson supported the understanding of half and quarter hours on the clock as well as the connection to partitioning into two and four equal parts. Each teacher saw the value in the hands-on learning and made connections between mathematics standards in the lesson.

Figure 5. Lesson on Telling Time

Implementing the job-embedded professional learning model has taught me a lot about supporting teachers as a mathematics coach. The teachers who have experienced at least two years of this professional learning model have become more comfortable with the process and are willing to reach out for planning and in-classroom support more regularly. This model builds the culture in a school building where everyone is a learner, we work together to meet students' needs, and collaboration among us is essential.

Summary

We as mathematics coaches value job-embedded professional learning for the following reasons:

- The structure gives us a better opportunity to build relationships with teachers and students in every class, in each grade-level within one given school year (we aim to be in the classroom 40% of the time).
- There is a higher probability of application of content or pedagogical knowledge because the mathematics coach is teaching alongside the teacher.

- There is sustained support for teachers and students and there are opportunities for collaboration throughout the school year.
- There is greater opportunity for immediate feedback. The mathematics coach can provide in-the-moment feedback while teaching with the teacher, as well as debrief with the teacher the day of the lesson.
- Teachers are more open to receive a coach's feedback because we have taught and learned with them and spent more than one day in their classroom.
- The teacher is more likely to reach out for other needs because we have built trust, they know that we understand their needs and their particular students, and they have co-taught with us.

It is our hope that by describing our experience with job-embedded professional learning, and through the perspectives of both teachers and administrators, mathematics coaches and teachers might be compelled to try this professional learning model.

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Go the Distance!

Jennieve Hill, Howard County Public School System

Abstract: *The last unit of many AP Calculus AB courses discusses area between two curves, volumes of revolution, and volumes by cross sections. Students particularly struggle with determining the length of the radii for volumes of revolution, especially when the axis of revolution is other than the x- or y-axis. To address these struggles, this activity focuses solely on determining a variety of distances as a one-day lesson prior to volume. Alternately, this activity may be done before discussing area between two curves.*

The last unit of many AP Calculus AB courses prior to the AP exam, which includes finding volumes of revolution, can be difficult for many students. It is a very conceptual area of Calculus.

During my first few years of teaching, I found that students struggled with determining the length of the radius/radii for volumes of revolution, especially when the axis of revolution is a line other than the *x*- or *y*-axis. Some students were uncomfortable with the idea of a distance not being a constant length. Some students also struggled understanding how and why the radius/radii changed if the axis of revolution is below (or to the left of) or above (or to the right of) the region being revolved.

Reflecting on this topic and the difficulties my students had, I decided to make an activity that focused solely on determining a variety of distances as a one-day lesson prior to beginning our discussion on volumes of revolution. If students had a day to work on finding different distances, both constant lengths and changing lengths, I figured it would make the task of identifying the radius/radii easier. The less new material all in one topic, the better! I tackled the concept of finding distances first, in order to directly apply that understand to finding the radius/radii during the next lesson of volumes of revolution.

My classroom is very collaborative, and I engage my students in discourse in both small groups and whole class frequently. At this point in the year, students are comfortable working together and/or asking for help. No two lessons are exactly the same, but here is one way I have successfully implemented this lesson.

Step 1. Spread 5 – 6 students out across the front of the room at various intervals where distances are already marked along the wall clearly for all to see.

Step 2. In groups (I seat students regularly in groups of 4-5), ask the students seated to determine how far apart several different pairs of students are from each other. Students find the task easy so it is low-risk for them to engage. Ask the class for volunteers willing to describe their thought process. This is a great time to implement the *Five Practices for Orchestrating Productive Mathematical Discussions*. Inevitably, a student (Student A) mentions they subtracted two values; Let this student know you are coming back to their idea shortly.

Step 3. Have the students standing in the front of the room go back to their seat and distribute the activity. The instructional routine of “Notice and Wonder” can be used well at this point. I require at least three ideas, and each idea must come from a different group of students. Using the instructional routine of Think-Pair-Share, have students work in their groups to determine the various distances in the first example, “Vertical Distances.” While students are talking and working together, write the letters A–H on the board. After students have sufficient time to think through this example, assign various letters to the groups of students. It is OK if a group has more than one letter for which they are responsible. The groups put their answer(s) for their assigned letter(s) on the board and discuss as a group the answers on the board. This is a great chance to facilitate whole class mathematical discourse!

Step 4. Repeat Step 3 using the second example, “Horizontal Distances.”

Step 5. After students have agreed that all the distances on the board are correct, select and sequence students to remind the class of the ideas shared during Step 2, drawing Student A back into the discussion last. Ask the class if this technique could be applied to the distances found in the first two examples, and if so, why? If not, why not? The instructional routine of Connect-Extend-Challenge is used in this part of the lesson. Careful facilitation of the discussion should lead students to develop a general formula/rule for finding any vertical or horizontal distance.

The two examples on the back, #1 and #2, can be assigned for homework or further classwork, depending on time. Like the two problems on the front, there are both constant and variable distances throughout each of the examples on the back.

I have used this activity for several years, and I have noticed improvement in my students' understanding. I have evaluated this improvement both informally (listening to class discourse through this activity and when discussing volumes of revolution) and formally (assessments). Class conversations have been more collaborative and formative and summative assessment scores that involve volumes of revolution have increased.

This is a great lesson to use during an evaluation as it will display most, if not all, of the Mathematical Teaching Practices for Exemplary Instruction as defined by the National Council of Teachers of Mathematics.

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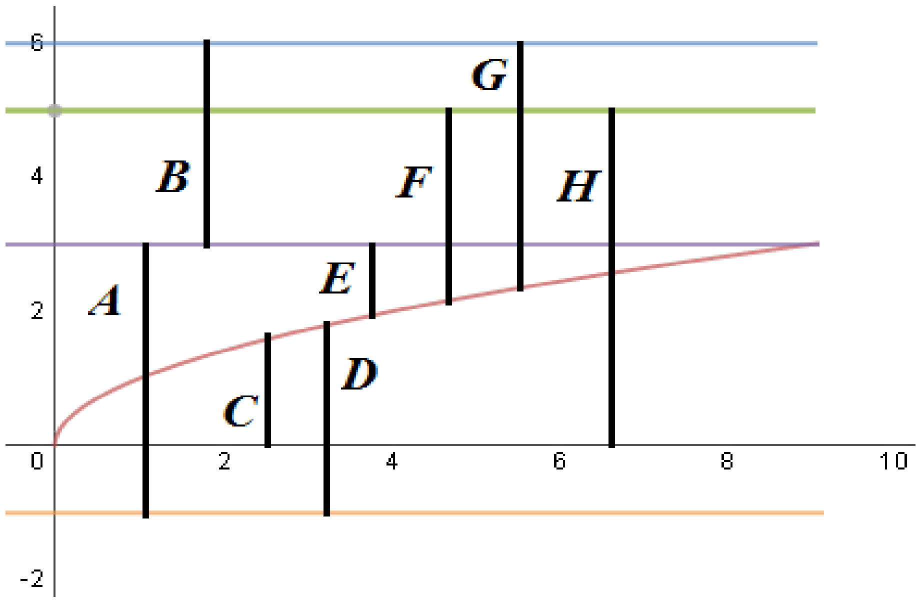
DISTANCES

Find the different length shown in the figures presented. The distances may or may not be dependent on the function(s) involved meaning some lengths may be constant and some may be a changing amount.

Vertical Distances

$y = \sqrt{x}$



A =

B =

C =

D =

E =

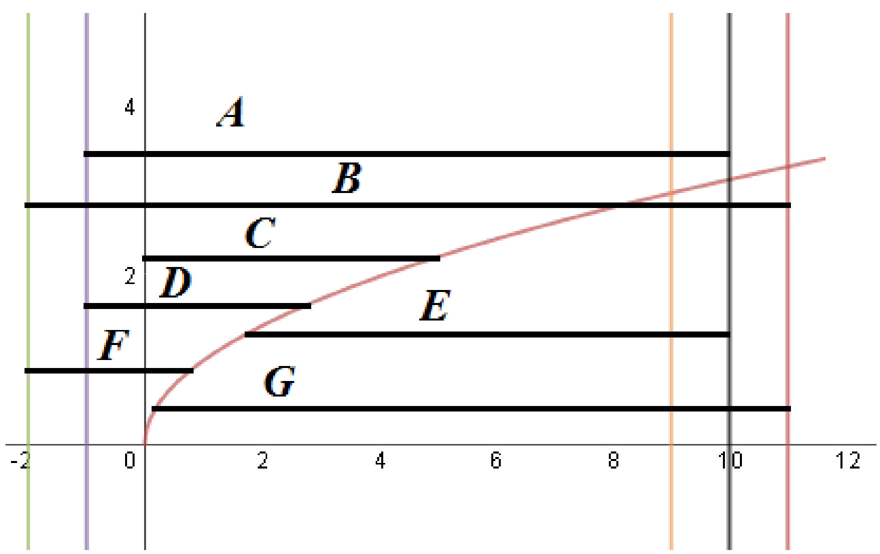
F =

G =

H =

Horizontal Distances

$y = \sqrt{x}$



A =

B =

C =

D =

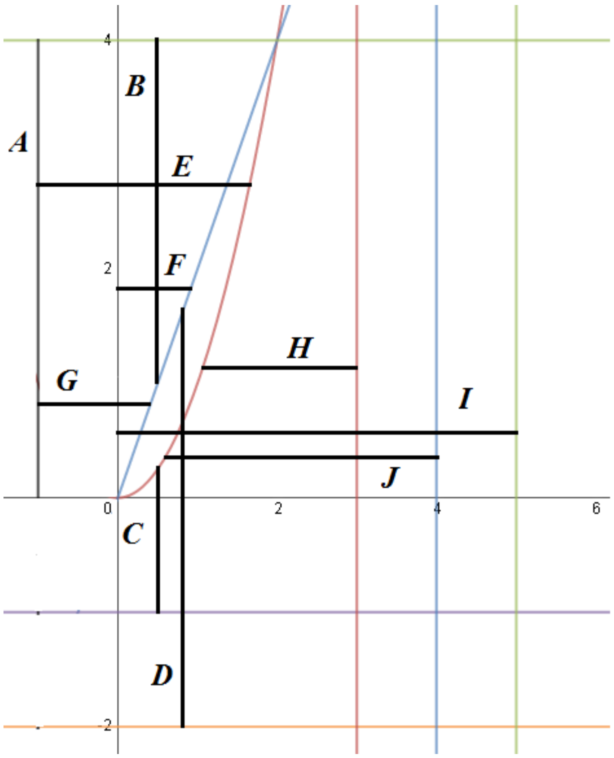
E =

F =

G =

Examples:

1. $y = x^2$ and $y = 2x$



A =

B =

C =

D =

E =

F =

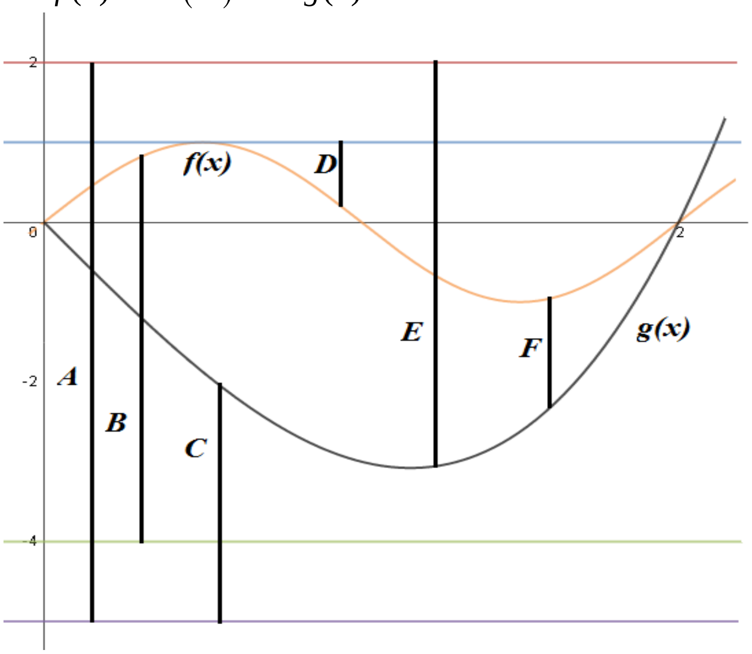
G =

H =

I =

J =

2. $f(x) = \sin(\pi x)$ and $g(x) = x^3 - 4x$



A =

B =

C =

D =

E =

F =

Answer Key:

Vertical Distances:

A = $3-(-1)=4$

B = $6-3=3$

C = $\sqrt{x}-0=\sqrt{x}$

D = $\sqrt{x}-(-1)=\sqrt{x}+1$

E = $3-\sqrt{x}$

F = $5-\sqrt{x}$

G = $6-\sqrt{x}$

H = $5-0=5$

Horizontal Distances:

A = $10-(-1)=11$

B = $11-(-2)=13$

C = $y^2-0=y^2$

D = $y^2-(-1)=y^2+1$

E = $10-y^2$

F = $y^2-(-2)=y^2+2$

G = $11-y^2$

Example #1:

A = $4-0=4$

B = $4-2x$

C = $x^2-(-1)=x^2+1$

D = $2x-(-2)=2x+2$

E = $\sqrt{y}-(-1)=\sqrt{y}+1$

F = $\frac{y}{2}-0=\frac{y}{2}$

G = $\frac{y}{2}-(-1)=\frac{y}{2}+1$

H = $3-\sqrt{y}$

I = $5-0=5$

J = $4-\sqrt{y}$

Example #2:

A = $2-(-5)=7$

B = $\sin(\pi x)-(-4)=\sin(\pi x)+4$

C = $x^3-4x-(-5)=x^3-4x+5$

D = $1-\sin(\pi x)$

E = $2-(x^3-4x)=2-x^3+4x$

F = $\sin(\pi x)-(x^3-4x)$
$=\sin(\pi x)-x^3+4x$