The Banneker Banner

Maryland Council of Teachers of Mathematics

The Official Journal of the Maryland Council of Teachers of Mathematics

Vol. 30, No. 2 **Spring 2017**

CONTENTS

Articles

EDITOR
Tricia K. Strickland, Ph.D.
Hood College
401 Rosemont Avenue
Frederick, MD 21701
301-696-3553
strickland@hood.edu

REVIEWERS

The individuals below have given their time and expertise to read and review manuscripts submitted for this edition of the *Banneker Banner*. We are very grateful for their help.

Andrew Bleichfeld, *MCTM*
Christy Graybeal, *Hood College*
Stewart Saphier, *MCTM*
Hanna Hager, *Montgomery County Public Schools*

Banneker Banner Submission Guidelines

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. To submit an article, please attach a Microsoft Word document to an email addressed to graybeal@hood.edu with "Banneker Banner Article Submission" in the subject line. Manuscripts should be original and may not be previously published or under review with other publications. However, published manuscripts may be submitted with written permission from the previous publisher. Manuscripts should be double-spaced, 12 point Times New Roman font, and a maximum of 8 pages. APA format should be used throughout the manuscript with references listed at the end. Figures, tables, and graphs should be embedded in the manuscript. As the Banner uses a blind review process, no author identification should appear on manuscripts. Please include a cover letter containing author(s) name(s) and contact information as well as a statement regarding the originality of the work and that the manuscript is not currently under review elsewhere (unless accompanied by permission from previous publisher). If electronic submission is not possible, please contact the editor to make other arrangements. You will receive confirmation of receipt of your article within a few days, and will hear about the status of your article as soon as possible. Articles are sent out to other mathematics educators for anonymous review, and this process often takes several months. If you have questions about the status of your article during this time, please feel free to contact the editor. Please note that photographs of students require signed releases to be published; if your article is accepted, a copy of the release will be sent to you and it will be your responsibility to get the appropriate signatures. If you would like a copy of this form at an earlier time, please contact the editor.

Readers who have developed successful classroom activities are encouraged to submit manuscripts in a format suitable for immediate use in the classroom. Submissions should help students understand mathematics or help teachers teach mathematics from either a conceptual or procedural instance while modeling effective pedagogy and addressing at least one Maryland College and Career Readiness Standards/Common Core Content and/or Practice Standards. A successful lesson or activity is one that is enjoyable to teach, that works well with students, that other teachers might adapt for use in their own classroom; and that is centered in developing students problem-solving and discourse

Student Activities can be for any grade level and should be in final format. Use one-inch margins, Time News Roman font, and a maximum of 4 pages. Authors should include a brief description of theirs and their students' experiences during the implementation as well as of insights gained from it. Any citation should follow APA Style (6th edition). The required list of references may not count toward the four-page limit of this section. Submissions will be sent out for peer review. Prospective authors should submit manuscripts of "Student Activities" to prof.lima@gmail.com .

Message from the Editor

Tricia K. Strickland

I am writing this message to inform readers that I will be stepping down as editor of the *Banneker Banner*. In the fall of 2012, I took over as editor of the *Banneker Banner,* and for the past 4.5 years, I have had the privileged of working with a wonderful group of mathematics educators from across the state of Maryland. The board members of the Maryland Council of Teachers of Mathematics (MCTM) are dedicated to the field of mathematics education and support Maryland mathematics teachers through the *Banner*, the annual conference, and special events throughout the year. Many thanks to our hard working MCTM board members! Please see page 43 for the list of MCTM board members.

I would also like to thank the many authors who have shared their expertise with us through their articles in the *Banner*. Authors voluntarily give many hours to produce articles that will enhance our understanding of a wide range of mathematical topics. I have learned much from their research and experiences. Please see page 2 and page 41 for information on submitting a manuscript for publication in the *Banneker Banner*.

It is time for a fresh perspective and we are fortunate to have Christy D. Graybeal, Ph.D. as the new editor of the *Banneker Banner*. Dr. Graybeal is the College Level Representative of the Maryland Council of Teachers of Mathematics and the former MCTM Newsletter Editor. She is also the founding president of the Association of Maryland Mathematics Teacher Educators, current secretary of the AMMTE, and an associate professor of education and mathematics at Hood College. There she teaches courses in mathematics education and mathematics at both the undergraduate and graduate levels. She received her undergraduate degree in elementary education and mathematics from Moravian College, her master's degree in mathematics from American University, and her doctorate in curriculum and instruction with a concentration in mathematics education from the University of Maryland. She is a former middle school mathematics teacher and is especially interested in fostering mathematical curiosity in students. The *Banneker Banner* is lucky to have Christy as the new Editor!

Elementary Mathematical Modeling: Get in the GAIMME

By Micah Stohlmann, Ph.D. University of Nevada, Las Vegas

Mathematical modeling has many benefits that make its implementation important. Through this approach students can develop mathematical understanding, see how mathematics is applicable to real life, and develop valuable 21^{st} century skills including communication and teamwork. Mathematical modeling allows for creativity, lets students make choices, and allows students to use their prior knowledge and strengths. Teachers can also use mathematical modeling as a formative assessment to inform and guide their instruction based on the understandings that students demonstrate.

The Common Core State Standards for Mathematics (CCSSM, 2010) include mathematical modeling as one of the eight Standards for Mathematical Practice (SMPs) as a proficiency that students should develop throughout their K-12 schooling. A benefit of mathematical modeling is that the implementation of mathematical modeling integrates other standards for mathematical practice as well. See Table 1.

With all of the benefits of mathematical modeling, the question becomes how can mathematical modeling be implemented effectively? The GAIMME (Guidelines for Assessment and Instruction in Mathematical Modeling Education) report provides useful information to answer this question.

Table 1.
Mathematical modeling integration with SMPs.

Standard for Mathematical Practice	**How it can occur in mathematical modeling**
1. Make sense of problems and persevere in solving them.	As students work through iterations of their solution, they continue to gain new insights into ways to use mathematics to develop their solution. The implementation structure of mathematical modeling allows students to stay engaged and to have sustained problem-solving experiences.
2. Reason abstractly and quantitatively	Mathematical modeling allows students to both contextualize, by focusing on the real-world context of the situation, and decontextualize, by representing a situation symbolically.
3. Construct viable arguments and critique the reasoning of others.	Careful reasoning and constructive critiquing are essential throughout mathematical modeling while groups are working and presenting their solutions.
5. Use appropriate tools strategically	Materials can be made available to groups as they work on mathematical modeling, including graph paper, calculators, computers, applets, dynamic software, and measuring devices.

(Stohlmann, Maiorca, & Olson, 2015)

The GAIMME report was published in 2016 and developed by leaders from the Society of Industrial and Applied Mathematics (SIAM) and Consortium for Mathematics and Its Applications (COMAP) with support by the National Council of Teachers of Mathematics. It provides help in the implementation of mathematical modeling from pre-kindergarten through college. The report was written because many teachers and students have had limited experience with mathematical modeling. Included in the report are example modeling problems, assessment resources, and information on what mathematical modeling is and isn't. The elementary grades writing team included Rachel Levy, Rose Mary Zbiek, Ben Galluzzo, and Mike Long.

The GAIMME report provides a definition of mathematical modeling. “Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena” (COMAP & SIAM, 2016, p. 10). To this definition we would add that, “mathematical modeling is an iterative process that involves open-ended, real world, practical problems that students make sense of with mathematics using assumptions, approximations, and multiple representations. Other sources of knowledge beside mathematics can be used as well” (Stohlmann & Albarracin, 2016, p.2).

Modeling Process

The modeling process provided in the GAIMME report is similar to the one provided in the CCSSM that also has six main components. As shown in Figure 1 the modeling process is not a linear or step-by-step process. Students may move back and forth between stages as they iterate to develop the best possible solution.

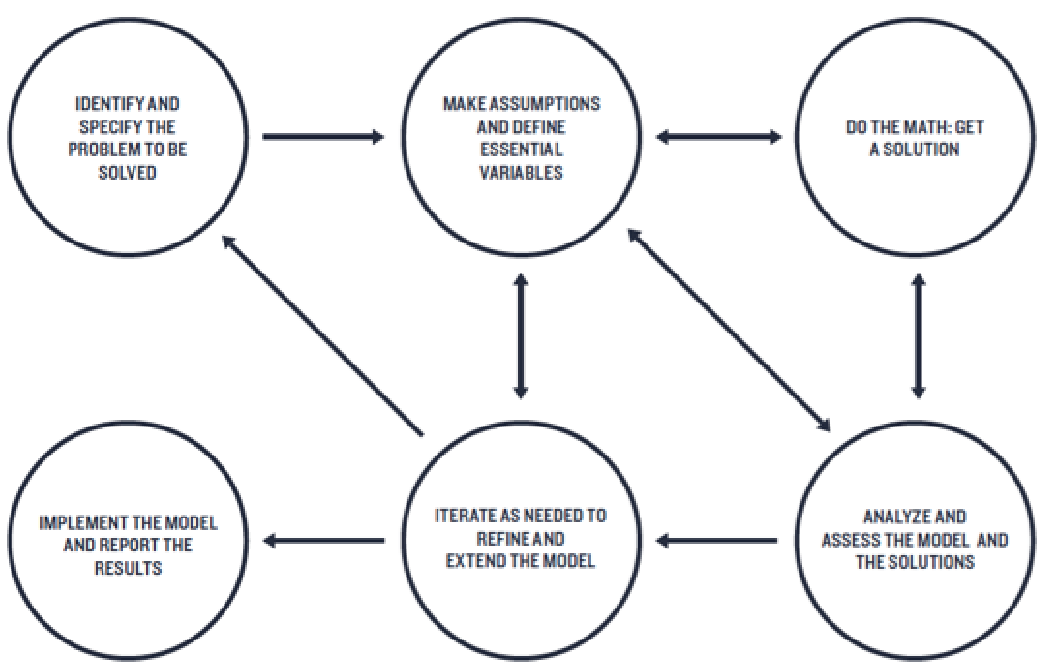


Figure 1. Modeling process (COMAP & SIAM, 2016, p.15)

To provide more detail on the components of the modeling process an example modeling problem provided in the GAIMME report will be discussed for grades 3 to 5. What should you bring for lunch? Though presented in a step-by-step fashion it is worth mentioning again that in practice students could move back and forth between stages.

Identify and specify the problem to be solved

This starting point is very different than a traditional textbook word problem in which all of the necessary information is given and there is a single, known, correct answer. There is refinement that takes place in understanding the real world situation and what the specific problem is that will be solved. In grades 3-5 the question could be refined to focus on, what is the best lunch?

Make assumptions and define variables

Students at this stage need to figure out what is most important. Things like type of foods, cost, and space available could be considered. Students could also take into account nutritional information and taste preferences. A definition of "best" would need to be created to determine how the best lunch is operationalized. Students may also have to decide how much space they have for a lunch if it is a cafeteria tray or a lunch made at home packed in a lunch box or bag.

Do the math: Get a solution

Throughout the modeling process multiple representations are important and should be encouraged. Students could use a bar graph to display the number of calories per food item or to display other nutrition information. A visual of how the lunch meets the five food groups could also be created with a drawing of a plate that shows the fractional part of the plate that correlates to the food groups. (https://www.choosemyplate.gov) Symbolic work could be shown that students describe based on the realistic situation that calculates total nutritional information for the lunch along with how the total cost was calculated.

Analyze and assess the model and solutions

This can be the hardest phase for students new to the modeling process as they may think that once they have an answer they are done. But it is vital to see how the model can be improved and if it satisfies the problem statement. Questions students can ask include: Do the results and math make sense in the realistic situation? How precise are the solutions? What would happen if we varied the assumptions such as a lunch box instead of a lunch bag?

Iterate as needed to refine and extend the model

The modeling process should involve evaluation of the limitations of the solution and how the solution can be strengthened. Students might find that the lunch they created is too expensive or that it tastes good but is not that healthy. Iterations can occur until a good balance is found and the best solution is developed given the constraints.

Implement the model and report results

At this point students are ready to share what they have created. They can state in what situations the model would or would not be useful and any assumptions that they made. After reporting their results it is good to give students further time to refine their models based on hearing and seeing what other groups have done.

Insights from implementation

In working with elementary teachers and students on mathematical modeling (e.g. Maiorca & Stohlmann, 2016; Stohlmann, Moore, & Cramer, 2013) we have come up with messages to reinforce with students or questions to ask at certain points of implementation (Table 2). If students have not done mathematical modeling before they need to be prepared for this type of experience. In implementing mathematical modeling, first students are given the problem statement and a whole class discussion can occur so that everyone understands what is being asked. Students then work in groups on creating their solution. After groups have been given time to work, it is good to randomly select one student in the group to present their solution. This is a cooperative learning strategy that helps to increase engagement and understanding. After presentations, groups then have a chance to revise their solutions based on what they have heard. Students individually can then reflect on how well they have worked in their groups and on how well they understood the mathematics.

Table 2.

Messages or questions for students when doing mathematical modeling

Before mathematical modeling
There is more than one right answer to this problem. There is not one type of person that is the best at mathematical modeling. Everyone can contribute. Make sure everyone in your group understands your solution. Use multiple ways to demonstrate your solution: pictures, graphs, symbols, words, or equations.
During mathematical modeling
Keep in mind what the problem is asking you to do. Make sure everyone in your group understands your solution. Does your solution make sense in the realistic situation? Can your solution be improved? Is your mathematics correct?
Before group presentations
Listen carefully to each group and think of a question to ask them. Try to see if there is anything from a group that you can use in your solution. Look to ensure that each groups' mathematics is correct.
After group presentations
After hearing from other groups' ideas, can our solution be improved? Is there any feedback we received to improve our solution? Was our solution clearly explained?
After mathematical modeling
What mathematics did my group use in our solution? How well did I understand the mathematics that was used? How well did I do working in the group?

The teacher as well can reflect after a mathematical modeling experience to see areas for improvement and to ensure that students were properly doing modeling.

-Did students start with a big, messy, real world problem?

-Did students ask questions and then make assumptions to define the problem?

-Did students identify what changes and what stays the same?

-Are students using mathematical tools to solve the problem?

-Are students communicating with someone who cares about the solution?

-Have students explained if/when their answers make sense?

-Have students tested their model/solution and revised if necessary? (IMMERSION, 2014)

Performance tasks on CCSSM assessments are similar to mathematical modeling problems so students need to be used to these experiences. The benefits of mathematical modeling are the real motivation for implementation; helping students develop robust mathematical knowledge and quality 21st century skills that will help them be successful no matter what career they end up pursuing. The final piece of advice from the GAIMME report is a valuable recommendation for teachers who have not implemented mathematical modeling before. "Start big, start small, just start" (COMAP & SIAM, 2016, p. 23).

References

Consortium for Mathematics and Its Applications (COMAP) & Society of Industrial and Applied Mathematics (SIAM). 2016. *Guidelines for Assessment and Instruction in Mathematical Modeling Education.* Bedford, MA & Philadelphia, PA: COMAP & SIAM.

Common Core State Standards Initiative (CCSSI). 2010. *Common Core State Standards for Mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Integrating Mathematical Modeling, Experiential learning and Research through a Sustainable Infrastructure (IMMERSION) and an Online Network for teachers in the elementary grades. 2014 – 2017. A National Science Foundation Sponsored STEM-C Program (NSF-1441024) through George Mason University, Montana State University, and Harvey Mudd College.

Maiorca, C. & Stohlmann, M. (2016). Inspiring students in integrated STEM education through modeling activities. In C. Hirsch & A.R. McDuffie (Eds.), *Annual Perspectives in Mathematics Education 2016: Mathematical Modeling and Modeling Mathematics.* (pp.153-161). Reston, VA: NCTM.

Stohlmann, M. & Albarracin, L. (2016). What is known about elementary grades mathematical modelling. *Education Research International, 2016*, 1-9.

Stohlmann, M., Maiorca, C., & Olson, T. (2015). Preservice secondary teachers' conceptions from a mathematical modeling activity and connections to the Common Core State Standards. *The Mathematics Educator Journal*, *24*(1), 21-43.

Stohlmann, M., Moore, T., & Cramer, K. (2013). Preservice elementary teachers' mathematical content knowledge from an integrated STEM modeling activity. *Journal of Mathematical Modelling and Application, 1*(8), 18-31.

The Mathematics of Driving

By Kristen Portalea, Montgomery County Public Schools

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

The demand for mathematics problems that capture students' attention and use their knowledge outside of school is increasing. While most Algebra students will not know how to drive, many are familiar with the road. Students see road signs, observe skid marks on the road, and hear about vehicle crashes on the news. After completing these lessons, students will be able to make mathematical connections to the signs, marks, and collisions they encounter and hear about every day. The car ride to school can be transformed into a learning opportunity. The grade of an upcoming hill can trigger students' knowledge of rate of change. The suggested speed for a turn can have students recalling that the curve is part of the circumference of a circle and measurements of the chord and middle ordinate can help determine the radius of that curve. Students' qualms about practical uses of their mathematical knowledge can be eased by highlighting the many careers that use mathematics to solve real world problems.

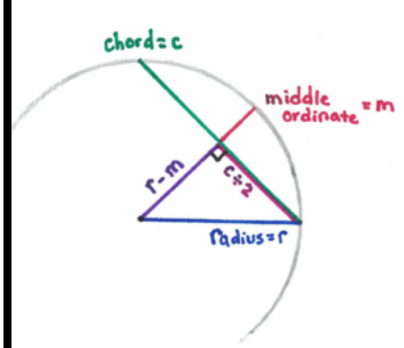
Nobody is above the physics and mathematics which govern the world we live in. The signs on a roadway warn drivers to stay within the physical limits of their cars and the road. The skid marks and debris in the roadway tell the story of drivers who exceeded these physical limits. The Mathematics of Driving Lesson Series leads students through the mathematics and basic physics used to place signage on roadways and investigate car crashes. The five lessons are appropriate for Algebra students and designed to give students the opportunity to connect what they have learned in mathematics classes to the real life field work of a police officer and an engineer.

In the first lesson of the series, armed with a level and tape measure, students venture outside to explore the rate of change or grade of a hill. Students use the level to find a point that is perfectly horizontal. Students then use the change in the horizontal distance (length of the level) and use a tape measure to find the change in the vertical distance (distance from the raised end of the level to the ground). Students then transition to finding the rate of change on a graph and eventually to finding the slope between two points. The lesson ends with students working as engineers with rate of change to determine the signs required by the Federal Highway Administration to warn drivers of an upcoming hill.

Later in the lesson, students also assume the job of a police officer. Many police departments have specialized collision reconstructionists to investigate the most serious vehicle collisions. Their job is "to explain how and why this wreck happened… performing a series of mathematical equations derived from physics" (Witt, 2016). One of the first things this unit does when it arrives at a crash scene is determine the coefficient of friction of the road. The coefficient of friction is a "dimensionless scalar value which describes the ratio of the force of friction between two bodies [Net Force] and the force pressing them together [weight]" (Montgomery County, Maryland – Department of Police, 2016). Beginning with Newton's Second Law, students manipulate equations until arriving at the coefficient of friction formula $= \frac{F}{w}$. Afterwards students collect data by pulling a make-shift drag sled. Students input their data to the formula to make observations of the impact of different surfaces and variables.

The task of solving a formula for a different variable becomes a necessary and interesting puzzle for students in the next lesson. Officers investigating a vehicle skidding to a stop have limited data to determine the speed a vehicle was traveling before skidding to a stop. To write a formula for the speed of the vehicle, students are only provided the variables immediately available to an officer at a crash scene – the length of the vehicle's skid marks (measured with a tape measure) and the coefficient of friction (measured by a drag sled). Students must use their knowledge of literal equations and algebraic properties to determine if a driver was speeding based on provided measurements.

Both traffic engineers and police officers calculate the critical speed of a curve, "a theoretical maximum vehicle speed on a roadway curve assuming the vehicle is tracking the exact radius of the roadway curve" (Glennon, 2016). How do you find the exact radius of a curve? The fourth lesson shows students how a curve in the road can be overlaid on part of a circle. Officers can measure a chord and middle ordinate (a perpendicular measurement from the middle of the chord to the curve) and then use the Pythagorean Theorem to determine the radius of the curve.



$$Leg^2 + Leg^2 = Hypotenuse^2$$

$$(r-m)^2 + \left(\frac{c}{2}\right)^2 = r^2$$

$$r^2 - 2mr + m^2 + \frac{c^2}{4} = r^2$$

$$-2mr + m^2 + \frac{c^2}{4} = 0$$

$$m^2 + \frac{c^2}{4} = 2mr$$

$$\frac{m^2 + \frac{c^2}{4}}{2m} = r$$

$$r = \frac{m}{2} + \frac{c^2}{8m}$$

Students also learn that "centripetal force is the force exerted by the vehicle holding you inside the vehicle and keeping your body moving in a circular path instead of flying in a straight line like it wants to" (Carrier, 2014, p. 142). In calculating the critical speed of a curve, students find where the centripetal force is equal to the lateral force of friction by working with literal equations. A vehicle breaking this balance, will likely crash if the centripetal force exceeds the lateral force (Carrier, 2014). Students put this knowledge to use as traffic engineers and must apply the Federal Highway Administration manual to determine what signs need to be used around the curve.

Vehicles with greater centripetal force than lateral force enter into a yaw. A yaw is when a vehicle rotates around its x-axis in a sideways motion, leaving distinct curved skid marks, called yaw marks, on the road (Glennon, 2016). Investigators have found that "a yaw most commonly occurs when a sudden steering input is applied to the vehicle. This can occur when a driver tries to negotiate a corner at too high a speed or when the driver oversteers a vehicle" (Carrier, 2014, p. 153). Similar to finding the critical speed of a curve, the yaw speed can be found by imagining the yaw marks as part of a circle. Chord and middle ordinate measurements are taken from the yaw marks and substituted into the same radius and speed equations used to find the critical speed of a curve (Carrier, 2014). The radius calculated is the radius of the travelling vehicle and the speed calculated is the speed at which the vehicle lost control and went into a yaw (Montgomery County, Maryland – Department of Police, 2016).

Equipped with the knowledge of a collision reconstruction detective, students are given pictures and measurements taken at an actual crash scene to apply their work. The Montgomery County Police investigated a collision in which a vehicle crashed into a curb. Upon arrival at the scene, police learned that the driver did not know what happened. The driver stated that she was driving the speed limit and following her GPS when it directed her to turn right. Officers at the scene recognized the tire marks left on the road as yaw marks and began to document the scene with pictures and the measurements needed for calculations. Just as the students practiced, a drag sled was used to find the coefficient of friction then chord and middle ordinate measurements were used to find the radius and critical speed of the curve as well as the radius and speed of the yaw. Students get to use the formulas they created and see the calculations work. The calculations represent the actual speed of a collision and are more valuable than theoretical calculations from a worksheet. It is an experience, using their knowledge to solve a problem that happened in real life.

The following is a preview of the culminating lesson. Remember that in the previous lessons students have discovered the skills to be successful at investigating this crash scene. The format has been modified to fit the constraints of this article. Teachers and parents can use The Mathematics of Driving Lesson Series as an opportunity to drive students' curiosity and understanding of why mathematics is so important in the world we live in. Remember, nobody is above the laws of physics and mathematics.

References:

Carrier, D. (2014). *Applied Physics in Policing.* Victoria, British Columbia, Canada: Friesen Press.

Glennon, J. C. (2016). *Glossary Of Terms For Crash Reconstruction*. Retrieved from Crash Forensics: http://www.crashforensics.com/crashreconstructionglossary.cfm

Montgomery County, Maryland - Department of Police. (2016). *Advanced Collision Investigation.*

Network ARC. (2016). *ARC Network Accident Reconstruction Directory*. Retrieved from Yaw Analysis for Accident Reconstruction: http://expertdirectory.arcnetwork.com/illinois/algonquin/photos/yaw-analysis-for-accident-reconstruction-8

Witt, A. (2016, May-June). Behind the Scenes with the Montgomery County Police Collision Reconstruction Unit. *Bethesda Magazine*. Retrieved from http://www.bethesdamagazine.com/Bethesda -Magazine/May-June-2016/Behind-the-Scenes-with-the-Montgomery-County-Police-Collision-Reconstruction-Unit/

Lesson 5 – Yaw

Introduction to a Yaw:

When a vehicle exceeds the critical speed of the turn, the driver loses control of the vehicle. "In terms of collision investigation, a yaw most commonly occurs when a sudden steering input is applied to the vehicle. This can occur when a driver tries to negotiate a corner at too high a speed or when the driver over steers a vehicle. In such cases, the vehicle starts to track through a turn, but the centripetal force exceeds the frictional forces" (Carrier, 2014, p. 153). If a vehicle yaws, there is typically a resulting crash.

When drivers travel at a safe speed around a curve, the rear tires track inside the front tires. However, when a vehicle enters into a yaw, the tires crossover and the rear tires track outside the front tires. The vehicle produces a specific kind of skid mark called yaw marks which are curved. This is where the investigators start their work.

The yaw marks, left by the tires during a yaw, become the circumference of a circle. A chord is then measured from the point after the tires crossed over to near the end of the yaw marks (preferably longer than 50 feet). The middle ordinate is measured perpendicularly from the middle of the chord to the yaw mark. Using the same formulas as calculating the critical speed of a curve, an investigator can determine the radius of the turn the driver made from the steering input and then the speed at which the vehicle lost control and entered a yaw.

Calculating a Yaw:

#1) A vehicle was driving quickly on a straight road and quickly cut the wheel. The car went into a yaw. How fast was the vehicle traveling before the driver lost control? The coefficient of friction of the road was 0.75.

Find the Radius of the Yaw Marks

$$r = \frac{m}{2} + \frac{c^2}{8m}$$

$$r = \frac{1.25}{2} + \frac{65^2}{8(1.25)}$$

$$r = 423.125\ feet$$

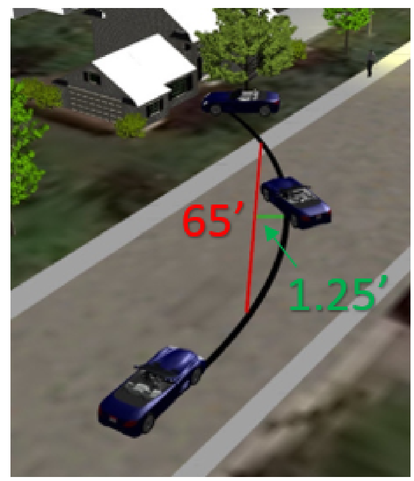
Find the Speed of the Yaw Marks

$$S = \sqrt{14.9569rf}$$

$$S = \sqrt{14.9569(423.125)(0.75)}$$

$$S = 68.89\ mph$$

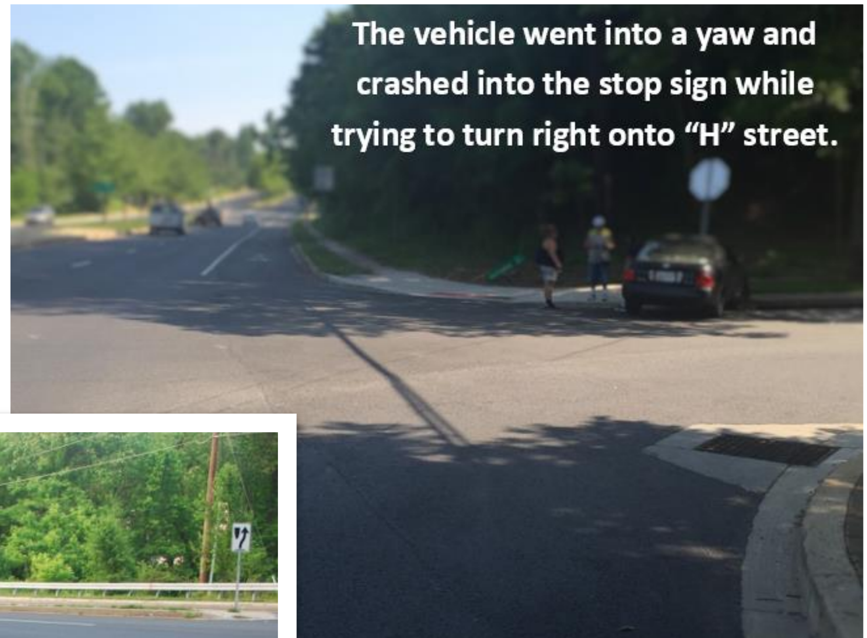
The vehicle was traveling at a speed of about 68.89 mph when the driver lost control.



http://expertdirectory.arcnetwork.com/illinois/algonquin/photos/yaw-analysis-for-accident-reconstruction-8

Culminating Activity:

#2) As a local police officer you are called to the scene of a vehicle collision. The young female driver was driving on a straight road, tried to turn down a side street but lost control of her vehicle and crashed. The driver said she does not know why but she suddenly lost control of her car. The speed limit on the road she was traveling is 40mph. Use the measurements given in the pictures and descriptions below in addition to calculations to write a police report detailing what happened in the crash. In your report include your calculations, an explanation of why the crash happened, and a diagram of the crash. Make your report detailed and thorough enough for others to understand.



The drag sled weighed 56.8 pounds and the force needed to pull it was 49 pounds.

$$f = \frac{F}{w}$$

$$f = \frac{49}{56.8}$$

$$f = 0.86$$

The coefficient of friction between the tire and the road was 0.86.

Radius of Curve:

$$r = \frac{m}{2} + \frac{c^2}{8m}$$

$$r = \frac{19}{2} + \frac{66^2}{8(19)}$$

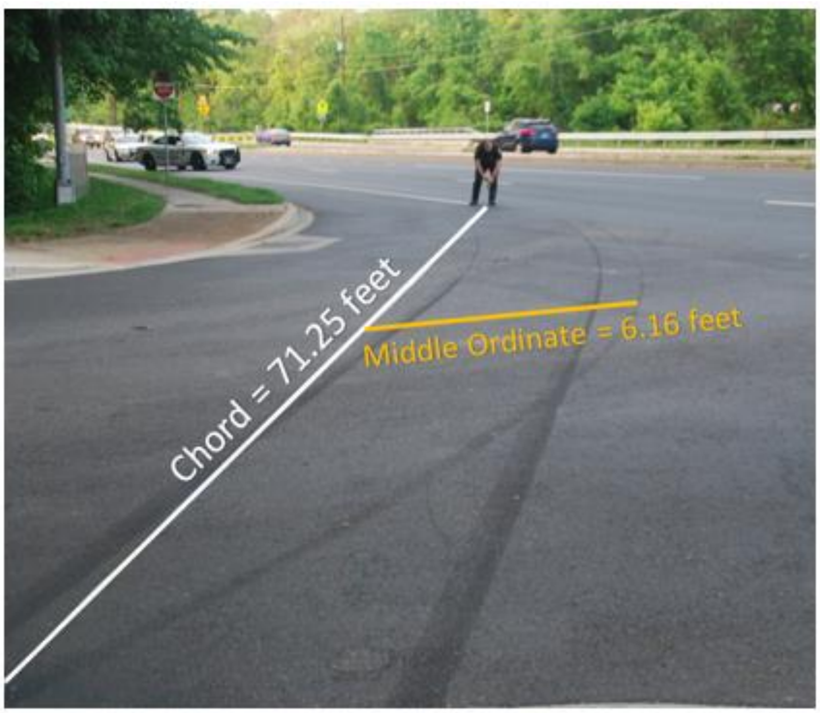
$$r = 38.16\ feet$$

Critical Speed of Curve:

$$S = \sqrt{14.9569rf}$$

$$S = \sqrt{14.9569(38.16)(0.86)}$$

$$S = 22.16\ mph$$



Radius of the Yaw:

$$r = \frac{m}{2} + \frac{c^2}{8m}$$

$$r = \frac{61.6}{2} + \frac{71.25^2}{8(6.16)}$$

$$r = 106.09\ feet$$

Speed of the Yaw:

$$S = \sqrt{14.9569rf}$$

$$S = \sqrt{14.9569(106.09)(0.86)}$$

$$S = 36.94\ mph$$

Sample Explanation:

The speed limit on "L" road is 40 miles per hour so the driver was not speeding when she began to turn on "H" street. However, the driver was exceeding the critical speed of the curve by about 14 miles per hour. By exceeding the critical speed of the curve, the centripetal force is greater than the frictional forces keeping the vehicle traveling its intended path and causing a crash.

A New Way of Thinking: Discrete Mathematics in High Schools

By Ann Eickelberg, Frederick County Public Schools

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

Currently, many public high school teachers in the United States find that there is little time or room for exploration of real-world topics and seem to leave the students asking the age old question "When will we need this in real life?" A Discrete Mathematics course in high school would allow students to still go through the curriculum, but explore topics that are more applicable to real-world scenarios.

The typical sequence of mathematics courses in high schools is Algebra I, Geometry, Algebra II, Pre-Calculus, and Calculus. Through personal experience, it is evident to me that students and teachers crave an innovative mathematics course that captivates and entices high school students. In my opinion, a possible approach to achieving this goal is by creating Discrete Mathematics as a high school course. Before we explore the benefits of Discrete Mathematics in high school, it is important that we know what Discrete Mathematics involves.

What is Discrete Mathematics?

"Discrete Math is the branch of mathematics dealing with objects that can assume only distinct, separated values. The term 'discrete mathematics' is therefore used in contrast with 'continuous mathematics,' which is the branch of mathematics dealing with objects that can vary smoothly (and which includes, for example, calculus). Whereas discrete objects can often be characterized by integers, continuous objects require real numbers" (Renze & Weisstein, 2016, p.

1). Small portions of Discrete Mathematics can be seen in everyday curricula from elementary school to high school, such as in arithmetic and geometric sequences and series and in probability, but unfortunately it is rarely seen as its own course.

Often times it is difficult to arrive at a simple definition for Discrete Mathematics. "'Discrete Math' is not the name of a branch of mathematics, like number theory, algebra, calculus, etc.

1. Rather, it is a description of a set of branches of math that all have in common the feature that they are "discrete" rather than "continuous."
2. The members of this set include (certain aspects of):
    - logic and Boolean algebra
    - set theory
    - relations and functions
    - sequences and series (or "sums")
    - algorithms and theory of computation
    - number theory
    - matrix theory
    - induction and recursion
    - counting and discrete probability
    - graph theory (including trees)"

(Rapaport, 2013, p. 1)

What are the benefits and risks of introducing a Discrete Mathematics course in high schools?

As a high school mathematics teacher, I am asked daily by students… "when will we need this in real life?" This simple question has encouraged me to include real-life examples in my daily lessons so that students can see the applications that they are seeking. During some topics, it is definitely challenging for teachers to find relevant, real-life scenarios that can illustrate the benefits of learning certain objectives.

Discrete mathematics has real-life applications built into each mathematics concept thereby making topics extremely relevant for everyday situations as well as college mathematics. This is seen in the article that states "discrete math, in particular counting and probability, allows students - even at the middle school level- to very quickly explore non-trivial "real world" problems that are challenging and interesting" (Patrick, 2016, p. 1).

Oftentimes the basic courses in high school - Algebra I, Geometry, Algebra II, Pre-Calculus, and Calculus- require that students memorize many formulas and then plug in numbers along the way to arrive at answers. The question in my mind that often arises is…. are the students truly understanding the problem and the formula(s) behind the solutions? Discrete mathematics gives students the ability to explore beyond the formulas and gives them an opportunity to create their own individual methods for discovery. It allows more room for hands-on work and problem based questions because it relates to real-life scenarios and can be often proven through visuals like Venn Diagrams. This tactile approach to learning will serve students well in future endeavors since hands-on and practical applications are what are used in the real world today.

"Algebra is often taught as a series of formulas and algorithms for students to memorize (for example, the quadratic formula, solving systems of linear equations by substitution, etc.), and geometry is often taught as a series of "definition-theorem-proof" exercises that are often done by rote (for example, the infamous "two-column proof"). While undoubtedly the subject matter being taught is important, the material (as least at the introductory level) does not lend itself to a great deal of creative mathematical thinking" (Patrick, 2016, p. 1). This shows how students might "pass" Algebra by plugging in formulas and memorizing equations, but not actually understanding the material. This issue can be resolved in a course like Discrete mathematics, which allows students to explore more topics on their own and develop theorems and proofs using this "hands-on" approach, as seen in the following quotation. Although all mathematics courses should be taught with this applicable and experimental approach, that is not always the case due to tight scheduling of state and county exams. "In contrast, with discrete mathematics, students will be thinking flexibly and creatively right out of the box. There are relatively few formulas to memorize; rather, there are a number of fundamental *concepts* to be mastered and applied in many different ways" (Patrick, 2016, p. 1). Some of these concepts include proofs using set theory, probability using combinations and permutations, and graph theory.

Discrete Mathematics is a course that will truly allow students to develop in all Standards of Mathematical Practice, especially reasoning abstractly and quantitatively and constructing viable arguments and critiquing the reasoning of others, which are often difficult ones to help students excel in (Common Core State Standards for Mathematics, 2010). This is evident in the quote from the Series in Discrete Mathematics and Theoretical Computer Science, which stated "discrete mathematics serves as an excellent vehicle for teaching students to communicate mathematically. Through carefully describing simple proofs and algorithms (e.g., instructions for building a Lego model), students acquire technical writing skills that will be useful in a variety of career and life situations" (Rosenstein, Franzblau, & Roberts, 1997, p. 18).

Although Algebra II, Pre-Calculus, and Calculus are fundamental in a student's ability to succeed in future college math courses, I truly believe that Discrete mathematics prepares students for the higher level courses and approaches in later years. It is the building block for theory courses like number theory and computer science courses like programming. Students who take Discrete math in high school would be better prepared to take the higher level courses earlier in college.

As the mathematics competition team coach at my high school, I also know the importance of Discrete mathematics in the high school and middle school contests. "Prominent math competitions such as MATHCOUNTS (at the middle school level) and the American Mathematics Competitions (at the high school level) feature discrete math questions as a significant portion of their contests. On harder high school contests, such as the AIME, the quantity of discrete mathematics is even larger. Students that do not have a discrete mathematics background will be at a significant disadvantage in these contests. In fact, one prominent MATHCOUNTS coach tells us that he spends nearly 50% of his preparation time with his students covering counting and probability topics, because of their importance in MATHCOUNTS contests" (Patrick, 2016). Students will be better prepared for these types of contests and competitions that can ultimately set them up for scholarships, internships, and other opportunities for the future.

Discrete math would only enrich the current curriculum because it would allow teachers to expand on topics that might not make sense without the prior knowledge of Algebra, Geometry, and even Statistics (which is offered as an elective at some schools). These topics can include certain aspects of graph theory when you are relating it to polygons in Geometry (such as vertices and edges), the Binomial Theorem, which is often a chapter that teachers cruise through in Algebra I without much explanation, and probability in Statistics and AP Statistics. Topics like the binomial theorem are often presented to Algebra students as a way to help memorize longer multiplication problems for polynomials, but Discrete mathematics could open the door for students to grasp a better understanding of the meaning, use, and reasoning behind the theorem. This would be beneficial even if it were taught after Algebra since the binomial theorem appears throughout various courses in mathematics, including Algebra 2, Pre-Calculus, Calculus, and Statistics. Discrete mathematics could help students comprehend proofs and formulas by connecting them to ideas learned in the class, such as Venn Diagrams, sets, and trees.

Of course with every advantage may come some disadvantages. I believe that the largest disadvantage would be fitting this course into a high school student's schedule without taking away one of the important core mathematics courses - Algebra, Geometry, Algebra 2, Pre-Calculus, and Calculus

How can Discrete math encourage a new way of thinking and incorporate real life applications?

I previously discussed that discrete math would involve a different approach to thinking and encourage students to explore outside the box. It allows students to get away from memorizing formulas and instead look at various solving methods - including modeling and proofs. A few examples of Discrete mathematics topics that can be used to develop a creative approach to math and relate the material to real life situations are described below:

- Spanning trees:

To grasp the definition of spanning trees, it is important to understand what a tree on a graph is. It is very relatable to what a tree is in everyday life, which makes it helpful for students to understand. A graph, made up of vertices and edges, is considered "a tree if and only if there is one and only one path joining any two of its vertices" (Asmerom, n.d., p. 1). "A spanning tree of a graph on n vertices is a subset of n-1 edges that form a tree" (Weisstein, Spanning Trees, 2016, p. 1). Spanning Trees are extremely useful in applications like algorithms and computer science. Students can connect the edges to everyday jobs, like engineers, cable workers, and construction workers. This is seen in careers where electrical grids are laid out for cost efficiency, cable lines are planned, and roads are built.

- Set Theory:

Set theory is "branch of mathematics that deals with the properties of well-defined collections of objects, which may or may not be of a mathematical nature, such as numbers or functions" (Enderton, 2016, p. 1) Set theory looks at elements, subsets, and sets and how they relate to each other. It explores different operations with the elements in sets including:

"Unary operation on a set is a function whose domain is that set. What distinguishes a unary operation from an ordinary function is the notation used, and often its relationships with other functions or operations. For example, the function that carries any real number x to the number $-x$ is a unary operation called negation. The range of the function is often the same set, but this is not required" (Erdelsky, 2010, p. 1).

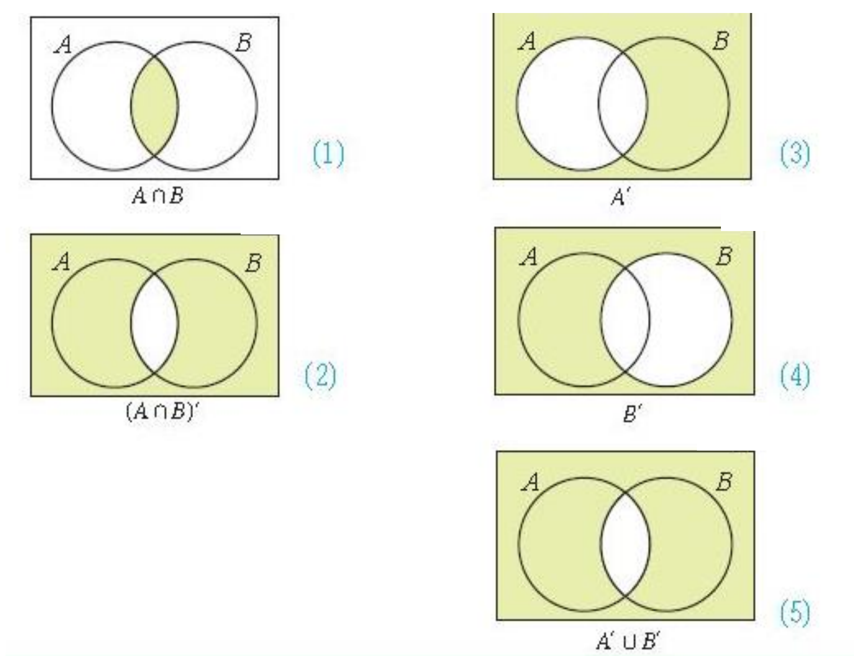
"Binary operation is a function whose domain is the cross product of two sets (or the cross product of a set with itself). For example, addition and multiplication are two binary operations on the set $R \times R$, where R is the set of real numbers. The image of an ordered pair (x,y) is usually written as $x+y$ for addition and xy for multiplication. Here x and y are called the operands. The former notation is usually used only for addition, or operations very much like addition. The latter notation is used for more general operations" (Erdelsky, 2010, p. 1).

With these operations, students can explore the properties, such as commutative and associative, and see if they hold. This type of discovery learning, where students can draw and collaborate to find creative approaches to proving theorems, prepares students to draw conclusions and create proofs for theorems like De Morgan's Laws. De Morgan's Laws state: "Let ∪ represent "or", ∩ represent "and", and ′ represent "not." Then, for two logical units" A and B,

$(A \cup B)' = A' \cap B'$ $\qquad\qquad$ $(A \cap B)' = A' \cup B'$

(Weisstein, de Morgan's Laws, 2016, p. 1)

Discrete mathematics allows students to visualize concepts, such as describing sets using Venn Diagrams, which makes it easier for some students to draw conclusions. Below is an example of how De Morgan's Law can be shown in Venn Diagrams.



(De Morgan's Laws, Venn Diagrams, Proofs Maths, Sets, 2011)

Discrete mathematics is a fun and exciting mathematics course that can give high school students a chance to explore advanced topics, understand meanings behind proofs, and capture their attention early on. It explores a wide variety of topics that can serve as building blocks for future courses in mathematics, science, and other disciplines. Overall, I truly believe that offering Discrete mathematics in high schools would benefit the students, at all levels, and bring "exploration" and "discovery" into the classroom.

References

Asmerom, G.A. (n.d.). Minimum spanning tree algorithms. N.p. Retrieved from http://www.people.vcu.edu/~gasmerom/MAT131/mst.html

Common core state standards for mathematics. (2010). Retrieved from http://www.corestandards.org/Math/Practice/

Cormen, T. H., Leiserson, C.E., and Rivest, R.L. Chapter 24: minimum spanning trees. 1990. Retrieved from http://staff.ustc.edu.cn/~csli/graduate/algorithms/book6/chap24.htm

De Morgan's laws, venn diagrams, proofs maths, sets. (2011). Retrieved from http://guideocom.blogspot.com/2011/08/de-morgans-laws-venn-diagrams-proofs.html

Discrete mathematics- spanning trees. (n.d.). Retrieved from http://staff.scem.uws.edu.au/cgi-bin/cgiwrap/zhuhan/dmath/dm_readall.cgi?page=17

Enderton, H. (2016). Set theory. *Encyclopedia Britannica*. Retrieved from https://www.britannica.com/topic/set-theory

Erdelsky, P.J. (2010). Elementary set theory. Retrieved from http://www.efgh.com/math/algebra/sets.htm

Patrick, D. (2016). Why discrete math is important. Retrieved from https://www.artofproblemsolving.com/articles/discrete-math

Rapaport, W.J. (2013). What is discrete math? Retrieved from http://www.cse.buffalo.edu/~rapaport/191/S09/whatisdiscmath.html

Renze, J. & Weisstein, E.W. (2016). Discrete mathematics. *MathWorld--A Wolfram Web Resource*. Retrieved from http://mathworld.wolfram.com/DiscreteMathematics.html

Rosenstein, J.G., Franzblau, D.S., & Roberts, F.S. Discrete mathematics in the schools. Vol. 36. Providence, RI: *American Mathematical Society, National Council of Teachers of Mathematics*, 1997. Print.

Weisstein, E.W. (2016). Spanning tree. *MathWorld--A Wolfram Web Resource*. Retrieved from http://mathworld.wolfram.com/SpanningTree.html

Weisstein, E.W. (2016). de Morgan's laws. *MathWorld--A Wolfram Web Resource*. Retrieved from http://mathworld.wolfram.com/deMorgansLaws.html

Preparing for AP Calculus in High School: The Ideal Precalculus Class

By Alison Blickenstaff, Frederick County Public Schools

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

As I was preparing to teach AP Calculus at the high school level for the first time, one of my colleagues mentioned that most of the prerequisite knowledge needed to be successful in AP Calculus was not taught in Precalculus, but in Algebra 2. As I embark on my fifth consecutive year of teaching AP Calculus AB, I have found this advice to be true. If that is the case, I wondered why it was even necessary for students to take Precalculus before AP Calculus. Was this the case for all Precalculus curricula across the nation, or the world? In this paper, I will discuss my findings on the prerequisites needed to take AP Calculus according to The College Board, as well as other important information about the course. I will then describe and analyze the Precalculus curricula from several online organizations and school systems around the nation, as well as one international curriculum. Finally, I will describe a revised Precalculus curriculum that would be ideal in order to fully prepare students to take AP Calculus in high school, a course that students do actually need before taking this rigorous college-level course.

AP Calculus Curriculum

The curriculum framework for AP Calculus AB and BC[1] is written by The College Board, "a mission-driven not-for-profit organization that connects students to college success and opportunity" (The College Board, 2016). College Board is the organization that writes the

[1] AP Calculus AB is equivalent to Calculus I in college, whereas AP Calculus AB and BC is equivalent to Calculus I and II.

curriculum for all Advanced Placement (AP) classes offered in the United States and their subsequent AP Exams, as well as other national tests such as SAT, PSAT, and NMSQT. In *Course and Exam Description: AP Calculus AB and AP Calculus BC* (The College Board, 2016), College Board gives the following description of the courses:

> Building enduring mathematical understanding requires students to understand the *why* and *how* of mathematics in addition to mastering the necessary procedures and skills. To foster this deeper level of learning, AP® Calculus is designed to develop mathematical knowledge conceptually, guiding students to connect topics and representations throughout each course and apply strategies and techniques to accurately solve diverse types of problems.

In this same document, College Board also discusses the prerequisites required for students upon entering AP Calculus. Students are expected to "complete the equivalent of four years of secondary mathematics designed for college-bound students: courses which should prepare them with a strong foundation in reasoning with algebraic symbols and working with algebraic structures." These courses should include the study of algebra, geometry and analytic geometry[2], trigonometry, and elementary functions. Within these broad topics, students should learn about the following types of functions: polynomial, rational, logarithmic, exponential, trigonometric, inverse trigonometric, and piecewise-defined. "In particular, before studying calculus[3], students must be familiar with the properties of functions, the composition of functions, the algebra of functions, and the graphs of functions." More specifically, College

[2] College Board did not distinguish between "geometry" and "analytic geometry;" they were listed as two separate mathematics domains, but no distinction was given between the two.
[3] The term "calculus" here refers to the mathematical domain, not the Calculus course. Some quoted material will not follow this convention.

Board expects students entering AP Calculus to understand the following concepts regarding functions: domain and range, even and odd, symmetry, and the behavior and aspects of functions and their graphs, including zeros, *y*-intercepts, and increasing and decreasing behavior. Furthermore, the unit circle and how the sine and cosine functions are related to it are concepts that students are expected to know. Students entering AP Calculus BC should have a basic understanding of polar coordinates equations, as well as sequences and series.

According to College Board, the three "big ideas" taught in AP Calculus are limits, derivatives, and integrals, and -- only in BC -- series. But according to well-known mathematics educators Joan Ferrini-Mundy and Karen Geuther Graham, there are actually four major areas involved in the study of calculus: functions, limits and continuity, derivatives, and integrals (Ferrini-Mundy & Graham, 1991). Based upon my experience teaching Calculus, I feel that the inclusion of this fourth big area of *function* is very appropriate, as students must have a firm foundation of understanding functions in order to be successful in using limits, derivatives, and integrals.

Based upon my experience and the use of my school district's chosen Calculus textbook, I have found that the philosophy of teaching calculus concepts graphically, numerically, and analytically is very useful in facilitating my students' gaining a well-rounded understanding of functions, limits, derivatives, and integrals (Edwards & Larson, 2011). For example, I have found that when students see a connection between a function's graph, its table of values, and manipulating the function algebraically, students develop a deeper understanding of the function and can see its association with other calculus concepts.

Student Misconceptions

There has been much research on common student misconceptions of calculus concepts. Two of the most common conceptual misunderstandings among Calculus students relate to functions and limits. Ferrini-Mundy and Graham found that students do not have a strong sense of the relationship between a function's graph and its formula; rather, they view a function's formula as disjoint from its graph (Ferrini-Mundy & Graham, 1991). Stemming from this misunderstanding, students often have a difficult time grasping the concept that a limit describes the behavior of a function near and around a point, but not necessarily at the point. Because there is a disconnect here, students then have further misunderstandings with the even more important and deeper calculus concept of derivative. This confusion all leads to the need of all mathematics curricula to stress the teaching of mathematical concepts in multiple representations, which is emphasized in the Common Core State Standards (CCSS) for Mathematics Standards for Mathematical Practices (SfMP) (Council of Chief State School Officers & National Governors Association, 2010). This will not only help to better prepare students for AP Calculus, but for future mathematics classes beyond Calculus. Mary E. Pilgrim, Assistant Professor of Mathematics at Colorado State University, writes that "Students who successfully complete Calculus should have a deep understanding of functions, the skills to analyze properties of functions using learn calculus techniques, and the ability to apply their learned knowledge to physical situations" (Pilgrim, 2014). In order to accomplish all of what the SfMPs and Pilgrim suggest, students need to have a stronger foundation in algebra and the multiple representations of functions before entering AP Calculus.

Examining Precalculus Curricula

Reviewing Precalculus Curricula

After researching current Precalculus curricula from various international, national, online, and local sources, I created the following table that summarizes necessary precalculus content for students to be successful in AP Calculus compared with what each Precalculus curriculum offers (International Baccalaureate Organization, 2012; Los Angeles Unified School District, 2014; New York State Education Department, 2013; California Department of Education, 2015; Department of Mathematics and Computer Science, 2013; Khan Academy, n.d.; International Academy of Science, 2014; Howard County Public Schools, 2015; Montgomery County Public Schools, 2016):

Necessary Precalculus Content	Yes	No	Unclear[4]
Algebra Review – Simplify Expressions, Solve Equations and Systems, Function Operations			
International Baccalaureate			X
Los Angeles Unified School District	X		
New York State Education Department		X	
California Department of Education	X		
Lehman College	X		
Khan Academy		X	
Acellus Academy		X	
Howard County (MD) Public Schools		X	
Montgomery County (MD) Public Schools		X	
Functions – Study of Polynomial & Rational Functions, Inverse Functions, Exponential & Logarithmic Functions			
International Baccalaureate			X
Los Angeles Unified School District			X
New York State Education Department		X	
California Department of Education			X
Lehman College	X		
Khan Academy		X	
Acellus Academy	X		
Howard County (MD) Public Schools			X
Montgomery County (MD) Public Schools	X		
Limits			
International Baccalaureate		X	
Los Angeles Unified School District		X	
New York State Education Department		X	
California Department of Education		X	

[4] "Unclear" indicates that the Precalculus course outline is not completely clear as to the depth that each concept is to be taught.

Lehman College		X	
Khan Academy		X	
Acellus Academy		X	
Howard County (MD) Public Schools	X		
Montgomery County (MD) Public Schools		X	
Trigonometry – Unit Circle, Solving Trig. Equations, Using Trig. Identities and Formulas, Graph and Analyze Trig Functions and their Transformations, Inverse Trig.			
International Baccalaureate			X
Los Angeles Unified School District	X		
New York State Education Department	X		
California Department of Education	X		
Lehman College		X	
Khan Academy	X		
Acellus Academy	X		
Howard County (MD) Public Schools	X		
Montgomery County (MD) Public Schools	X		
Calculus BC Topics – Sequences and Series, Conics, Parametric Equations, Polar Coordinates			
International Baccalaureate		X	
Los Angeles Unified School District		X	
New York State Education Department		X	
California Department of Education		X	
Lehman College		X	
Khan Academy		X	
Acellus Academy			X
Howard County (MD) Public Schools	X		
Montgomery County (MD) Public Schools		X	
Emphasize Multiple Representations of Functions			
International Baccalaureate			X
Los Angeles Unified School District	X		
New York State Education Department			X
California Department of Education			X
Lehman College			X
Khan Academy			X
Acellus Academy			X
Howard County (MD) Public Schools			X
Montgomery County (MD) Public Schools			X

As can be seen from the various Precalculus curricula, there are many overlaps between major units and subtopics among them all. The most common similarities include some algebra review, some study of functions, and the study of trigonometry. Other common similarities not listed in the table include geometry, complex numbers and polar coordinates, vectors and parametric equations, conics, and topics from discrete mathematics and probability and statistics. Most of the reviewed curricula emphasize the importance of being knowledgeable about all of the different types of functions students have learned about in previous Algebra classes, but some also emphasize the importance of knowing the graphs of each type of function and the manipulation of these functions using different operations. Only one source includes an introduction to the concept of limit (Howard County Public Schools, 2015), which is an important and frequently misunderstood topic for many Calculus students still today (Ferrini-Mundy & Graham, 1991). Furthermore, there are very few marks in the "yes" column, showing that many of the researched curricula may not be adequately preparing their students to be successful in AP Calculus.

My Changes to the Precalculus Curriculum

Now that the necessary prerequisite knowledge and common student misunderstandings for AP Calculus have been addressed, in addition to the discussion of many current Precalculus curricula units and topics, I will now discuss my proposed revisions to the current Precalculus curriculum in order to make the ideal course designed to better prepare students to be successful in taking AP Calculus at the high school level. But first, I will take one more look at research on the Precalculus curriculum from a Professor of Mathematics from the University of Wisconsin – Madison. In his article, "Precalculus Mathematics: A Look Through the Big End of the Telescope," John G. Harvey describes the following objectives to be taught in the Precalculus curriculum during the late 1970's, which are still very applicable today: number systems, functions and relations, polynomial functions with real coefficients, rational functions, trigonometric and inverse functions, trigonometric graphs and identities, logarithmic and exponential functions, and analytic geometry and linear algebra (Harvey, 1978). Whereas many of the current international, national, online, and local Precalculus curricula reviewed include many of these topics, some of these essential topics are not included. Many of them also include additional topics that are not necessary in order for students to be adequately prepared for AP Calculus.

The curriculum that I have created covers all of the topics that Harvey outlines and will incorporate topics and a review of skills that align with the prerequisite knowledge as required by College Board, in order to better prepare students to avoid the common misunderstandings of today's Calculus students. My curriculum focuses on a fine-tuning of basic algebra skills needed to solve many calculus problems, including factoring, and simplifying expressions involving fractions, as well as properties of exponential and logarithmic functions, performing operations with functions, and solving different types of equations with and without a calculator. It includes an extensive study of the different families of functions, which students review through graphical, numerical, and analytical approaches in order to understand multiple representations. Students will also re-examine the following "parent functions": polynomial, rational, radical, absolute value, exponential, and logarithmic. They will explore transformations, analysis of functions, piecewise-defined functions, and inverse functions at a greater depth than in previous courses. Students will begin to use sign charts (number lines that incorporate the sign of the function on specific intervals) to graph and analyze function behavior, as it is a skill used in AP Calculus within multiple units. A unit on limits is incorporated early in the curriculum so that students can begin forming their understanding of this concept, and so they can apply their knowledge of and manipulation of functions in order to evaluate limits graphically, numerically, and analytically. It includes a study of trigonometry, but only those concepts and skills needed in AP Calculus: graphs of functions, the unit circle, solving trigonometric equations, basic trigonometric identities, inverse trigonometry, and application problems. Finally, a unit including a study of sequences, series, vectors, and conics ends the course, as these topics will be an introduction to topics taught in AP Calculus BC only. Overall, my Precalculus course focuses on students' having a more in-depth understanding of functions in multiple representations. Students will have a higher skill-level with the algebra behind function operations and evaluating expressions. They will be exposed to the concept of limit earlier and in connection with functions they have just recently studied. The graphing calculator will also be incorporated into every unit so that students will know exactly what calculator skills they must know, as well as which types of questions they are expected to answer with or without a calculator. My hope is that my Precalculus course will be structured like an AP Calculus course: fast-paced and rigorous, one where the common misconceptions of current Calculus students can be eliminated, and one that students do actually need in order to be successful in a high school AP Calculus class.

If you are interesting in learning more about my Precalculus curriculum, you can contact me at alison.blickenst@fcps.org.

References

Acellus Academy. (2016). *About acellus academy: A program of the international academy of science*. Retrieved from https://www.acellus.com/courses/math-pre-calculus.php.

California Department of Education. (2015). *Precalculus chapter of the mathematics framework for California public schools: Kindergarten through grade twelve*. Retrieved August 31, 2016, from California Department of Education Web site: http://www.cde.ca.gov/ci/ma/cf/documents/mathfwprecalculus.pdf.

Department of Mathematics and Computer Science, Lehman College, City University of New York. (2013). *Precalculus syllabus*. Retrieved October 24, 2016 from Lehman College Web site: http://lehman.edu/academics/mathematics-computer-science/documents/172-Syllabus-2013.pdf.

Edwards, B.H., & Larson, R. (2011). *Calculus of a single variable: Early transcendental functions* (5th ed.). Boston: Brooks/Cole CENGAGE Learning.

Ferrini-Mundy, J., & Graham, K.G. (1991). An overview of the calculus curriculum reform effort: Issues for learning, teaching, and curriculum development. *The American Mathematical Monthly, 98*(7), 627-635.

Harvey, J.G. (1978). Precalculus mathematics: a look through the big end of the telescope. *The Mathematics Teacher*, *71*(1), 21-28.

Howard County Public Schools, Office of Secondary Mathematics. (2015). *Pre-calculus GT essential curriculum 2015-2016*. Retrieved September 3, 2016, from Howard County Public School System Web site: http://www.hcpss.org/f/academics/math/pre-calc-gt-curriculum.pdf.

International Academy of Science. (2014). International academy of science pre-calculus course curriculum. Retrieved September 2, 2016, from Acellus Academy Web site: http://www.science.edu/Acellus/curriculum/mathematics/lesson-lists/Pre-Calculus.pdf.

International Baccalaureate Organization. (2012). *Diploma programme mathematics SL guide*. Retrieved September 2, 2016, from International Baccalaureate Web site: http://www.ibo.org/programmes/diploma-programme/curriculum/mathematics/.

Khan Academy. (n.d.). *Precalculus*. Retrieved August 31, 2016, from Khan Academy Web site: https://www.khanacademy.org.

Los Angeles Unified School District Office of Curriculum Instruction and School Support. (2014). *Common core precalculus curriculum map 2016-2017*. Retrieved August 31, 2016, from Los Angeles Unified School District Web site: http://achieve.lausd.net/Page/6083.

Montgomery County Public Schools. (2016). *Precalculus/honors precalculus*. Retrieved August 31, 2016, from Montgomery County Public Schools Web site: http://www.montgomeryschoolsmd.org/curriculum/math/high/precalculus/.

New York State Education Department. (2013). *Precalculus and advanced topics*. Retrieved August 31, 2016, from Engage NY, New Work State Education Department Web site: https://www.engageny.org/content/precalculus-and-advanced-topics.

Pilgrim, M.E. (2014). Addressing the standards for mathematical practice in a calculus class. *The Mathematics Teacher*, *108*(1), 52-57.

The College Board. (2016). *Course and exam description: AP calculus AB and AP calculus BC*. New York: The College Board.

The Banneker Banner

Maryland Council of Teachers of Mathematics

OPEN CALL

OPEN CALL manuscripts are focused on a particular theme or have a particular format. Open Call manuscripts do not have a submission deadline. It is a great way to begin sharing your ideas professionally. Themes for Open Call manuscripts include:

ACTIVITIES FOR STUDENTS - Manuscripts describe student-centered activities suitable for immediate classroom use in grades Pre-K – 12. Authors share teacher implementation tips, differentiation options, and assessment strategies to promote discussion about important concepts, connections, and representations of the mathematical ideas central to a class or course.

CONNECTING RESEARCH TO TEACHING - Manuscripts describe lessons learned from research that informs classroom practice in a manner appropriate for teachers' application to classroom practice or otherwise suitable for reflective discussions at department meetings or other gatherings.

LEARNING WITH TECHNOLOGY - Manuscripts describe teachers' experiences and reflections on the meaningful use of technology to enhance instruction, assessment, or curriculum and how the use of technology has changed the teaching and learning of mathematics.

MATHEMATICS TEACHING IN PreK - 12 - Manuscripts describe teachers' experiences bringing the mathematics curriculum to life in PreK to 12 classrooms. Teachers share their experiences and reflect on the successes and challenges of engaging young learners in the learning of mathematics.

TEACHING MATHEMATICS TO DIVERSE LEARNERS- Manuscripts describe teachers' experiences adapting mathematics curricula for diverse learners, such as Students with Disabilities, English Language Learners, and Gifted and Talented students. Teachers share their instructional practices and reflect on the successes and challenges of supporting diverse learners to reach their fullest potential as mathematics learners.

See our website for additional details:
https://www.marylandmathematics.org/banneker-banner.html

MEMBERSHIP
The Maryland Council of Teachers of Mathematics (MCTM) is the professional organization for Maryland's teachers of mathematics. Our members represent all levels of mathematics educators, from preschool through college. We are an affiliate of the National Council of Teachers of Mathematics (NCTM). Our goal is to support teachers in their professional endeavors and help them to become agents of change in mathematics education.

First Name: ______________________ Last Name: _________________________

Home Address: ____

City: ________________________ State: ________ Zip: _____________________

Home Phone: ________________ E-mail address: __________________________
MCTM does not share email addresses

School System or Affiliation: ____

Level: Check applicable categories:
__ Early Childhood (Pre-Kindergarten to Grade 2)
__ Elementary (Grades 3 – 5)
__ Middle School (Grades 6 – 8)
__ High School (Grades 9 – 12)
__ Higher Education (Grade 13+)

Signature: _______________________________________ Date:____________

Join on-line at https://www.marylandmathematics.org/or mail a $15 check, made out to MCTM, and this completed form to:

Mrs. Holly Cheung
Treasure
Howard County Public School System
10910 Clarksville Pike
Ellicott City, MD 21042

MCTM Board of Directors

The MCTM Board of Directors and Committee Chairs are here to serve you. To contact a member of the board or committee, click on the e-mail link provided.

President-Elect	Jennifer Novak	jennynovak89@gmail.com
Past President	Andrew Bleichfeld	Andrew.bleichfeld@hcps.org
Executive Director	Luis Lima	Prof.Lima@gmail.com
Recording Secretary	Ming Tomayko	mtomayki@towson.edu
Treasurer	Holly Cheung	Holly_Cheung@hcpss.org
Central Region Rep	Sue Thompson	sthompson@bcps.org
Western Region Rep	Scott Trexler	setrexl@carrollk12.org
Southern Region Rep	Marcie Sun	Marcia.frank@pgcps.org
Eastern Region Rep	Jeanine Brizendine	jbrizendine@gmail.com
Early Childhood Rep	Mark Zimmerman	mzimmerman@bcps.org
Intermediate Rep	Verna Washington	Verna_L_washington@mcpsmd.org
Middle School Rep	Babette Margolies	babsmar@gmail.com
High School Rep	John Fontinell	jfontinell@bcps.org
Preservice Rep	Kourtney Harrison	Kharri40@students.towson.edu
College Rep	Christy Graybeal	graybeal@hood.edu
Member-at-Large	Stacy Shaener	sshaener@verizon.net
Membership Chair	Matthew Cox	Matthew_cox@hcpss.org
Nominations Chair	Camille Tomlin	Camtom47@hotmail.com
New Teacher Packets Chair	TBD	TDB
Awards Chair	Jeanine Brizendine	jbrizendine@aacps.org
Math Month Chair	Bonnie Yost	byost@bcps.org
Mathalon Chair	Stew Saphier	stewsaphier@gmail.com
Webmaster	Mike Barnhart	mbarnhart@aacps.org
Newsletter	Francine Johnson	johnsonf@hood.edu
Banner Editor	Tricia Strickland	Strickland@hood.edu
Historian	Babette Margolies	babsmar@gmail.com
NCTM Liaison	Bill Barnes	William_barnes@hcpss.org
Government Relations	Sue Vohrer	svohrer@aacps.org
MSDE Representative	Linda Schoenbrodt	linda.schoenbrodt@maryland.gov
MCTM Conference Chairs	John Fontinell,	jfontinell@bcps.org
	Kevin Wajek	kwajek@aacps.org

The Maryland Council of Teachers of Mathematics is an affiliate of the National Council of Teachers of Mathematics. Membership in the MCTM is open to all persons with an interest in mathematics education in the state of Maryland. To become an MCTM member, please visit our website: https://www.marylandmathematics.org/.

Furthermore, the MCTM Board invites all members to become actively involved in our organization. To become involved, please contact one of the officers listed above. We would love to hear from you!

Maryland Council of Teachers of Mathematics
c/o Holly Cheung
Treasure
Howard County Public School System
10910 Clarksville Pike
Ellicott City, MD 21042

MCTM Mission Statement: The MCTM is a public voice of mathematics education, inspiring vision, providing leadership, offering professional development, and supporting equitable mathematics learning of the highest quality for all students.