The Banneker Banner

Maryland Council of Teachers of Mathematics

The Official Journal of the Maryland Council of Teachers of Mathematics

Vol. 29, No. 1 Spring 2014

CONTENTS

Articles

EDITOR
Tricia K. Strickland, Ph.D.
Hood College
401 Rosemont Avenue
Frederick, MD 21701
301-696-3553
strickland@hood.edu

REVIEWERS

The individuals below have given their time and expertise to read and review manuscripts submitted for this edition of the *Banneker Banner*. We are very grateful for their help.

William Barnes, *Howard County Public Schools*
Andrew Bleichfeld, *Harford County Public Schools*
Christy Graybeal, *Hood College*
Luis Lima, *Baltimore City Public Schools*
Ming Tomayko, *Towson University*
Melissa Waggoner, *Howard County Public Schools*

Many thanks to Andrew Bleichfeld and Edward Nolan for their assistance with editing this issue.

Banneker Banner Submission Guidelines

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. To submit an article, please attach a Microsoft Word document to an email addressed to strickland@hood.edu with "Banneker Banner Article Submission" in the subject line. Manuscripts should be original and may not be previously published or under review with other publications. However, published manuscripts may be submitted with written permission from the previous publisher. Manuscripts should be double-spaced, 12 point Times New Roman font, and a maximum of 8 pages. APA format should be used throughout the manuscript with references listed at the end. Figures, tables, and graphs should be embedded in the manuscript. As the Banner uses a blind review process, no author identification should appear on manuscripts. Please include a cover letter containing author(s) name(s) and contact information as well as a statement regarding the originality of the work and that the manuscript is not currently under review elsewhere (unless accompanied by permission from previous publisher). If electronic submission is not possible, please contact the editor to make other arrangements. You will receive confirmation of receipt of your article within a few days, and will hear about the status of your article as soon as possible. Articles are sent out to other mathematics educators for anonymous review, and this process often takes several months. If you have questions about the status of your article during this time, please feel free to contact the editor. Please note that photographs of students require signed releases to be published; if your article is accepted, a copy of the release will be sent to you and it will be your responsibility to get the appropriate signatures. If you would like a copy of this form at an earlier time, please contact the editor.

The Gavel

Would you go into teaching again if you had the chance?

Andrew Bleichfeld, MCTM President

I love teaching. I love my classes, I love my curriculum, I love my students, I love my school. But some of the extras that go along with the teaching are starting to make me wonder if I would choose teaching again if I had it to do all over.

Where were "essential questions" twenty years ago? Where was "data collection" five years ago? Where were "student learning objectives" just two years ago? Have these made teaching better. Most definitely. Have they made it more enjoyable? Probably not.

So my conflict comes because I am sponsoring a student from a community college for the next few weeks. This 22-year old mentee has to observe thirty hours of classroom instruction and has to teach two lessons, either to an entire class or just to a smaller group of class members. Through this course and these observations, he can then develop an opinion about whether he wants to pursue teaching as a career.

I am trying to stay positive with this student. I am trying to show him how enjoyable teaching can be. And lucky for him, I am shielding him from the paperwork aspect of the job. He doesn't see all of the planning and preparation that goes into my lessons. He doesn't see all of the grading that goes on. He doesn't see the comparisons of data for each student. He doesn't see the SLO's I have made and the efforts I have to make to ensure that my students will reach the stated level of performance. He doesn't see the questions I ask myself about whether I am requiring students to perform higher-level thinking often enough. He doesn't see the worry that I possess about upcoming HSA, MSA, or PARCC exams.

But should I be showing him "the dark side" of teaching. Should he see the tremendous amount of effort that good teaching involves?

My brother knows that he can get me riled up by just uttering the phrase "Well, you get the whole summer off." I work hard during the school year. The day is rare when I work a contracted day from 7 AM to 2:20 PM. I am always working 10 or 12 hour days to ensure that I am prepared and to ensure that my students are successful. Very often I have brought home papers to grade. Very often I have planned lessons at home on my own time.

So part of me looks at my mentee and makes me wonder that if I were his age, would I step into teaching again? For the sake of the profession, for the sake of our students, I hope that the answer is yes.

The following quote is from a University of North Caroline teaching education website, http://www.unc.edu/uncbest/teacher.html .

> "For many people, their work is a means to an end. They work for a paycheck in order to live their lives. But those called to teach have a true vocation. To those with whom you interact most during your day of teaching - the students - you are not an employee but a friend, a mentor and a guide to the world. A teacher makes a difference in the world by enabling each of his or her students to fully maximize their talents, imagination, skills and character."

So, would I step into teaching again if I had it to do all over again? I know that the answer is yes.

Puzzler

http://mathforum.org/rec_puzzles_archive/geometry.html

Can three houses be connected to three utilities without the pipes crossing?

Answer and explanation on p.25

Source of free icons: http://findicons.com/search/house

Deep Conceptual Understanding & the Use of Manipulatives in Secondary School Mathematics

Luis Lima
Baltimore City Public Schools

A number is an idea, an abstraction. No one has ever seen a number and no one ever will. We see illustrations of this idea, but not the idea itself. The symbol "3" is used to bring forth a series of experiences and a set of memories that we have collected involving the concept of three; but the symbol 3 in and of itself is not the concept. Therefore, how do we teach children about the concept of number if it is a total abstraction?

The answer revolves around the notion of isomorphism. Derived from the Greek *iso*, meaning "equal", and *morphosis*, meaning "to form" or "to shape"; isomorphism refers to the identification and use of a parallel structure that has the same properties as those of the abstraction of number and yet is more accessible and manipulable. These interpretations or embodiments of a concept can be operated on in order to help make conclusions about the more abstract system of numbers. Similarly, manipulative materials can be used as isomorphic structures to represent more abstract mathematical notions we want students to learn (Post, 1981). In other words, manipulative materials provide students and teachers physical models with which they can interact and which help them create mental models to make sense of abstract mathematical ideas (Jones, Uribe-Florez, Wilkins, 2011).

> ***Common Core State Standards of Mathematics***
> ***Shift 4: Deep Conceptual Understanding***
>
> **Deep conceptual understanding of core content at each grade is critical for student success in subsequent years. Students with conceptual understanding know more than isolated facts and methods - they understand why a mathematical idea is important and the contexts which it is useful. Teachers take time to understand the Standards for Mathematical Practice that describe the student expertise needed to develop a deep conceptual understanding of mathematics (NRC, 2001, p. 118; CCSSM, 2010, p. 4, 6-8).**

Why use manipulatives to teach Secondary School Mathematics?

Students come to the classroom with varying knowledge, life experiences and backgrounds. A key component in successfully developing mathematical competence in students with diverse backgrounds is to help them construct meaning of the mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. In this progression, the use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages, and enhance the formation of sound, and transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Moreover, meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics to further solidify these mathematical ideas (Bruner, 1966; Burns, 2007; Sowell, 1989).

Concurrently, research strongly suggests that students gain better conceptual understanding and are more successful in demonstrating mastery of new mathematical ideas when they have a chance to experience mathematical concepts using manipulative materials over a sustained period of time (Frei, 2008, p. 101). However, many teachers, especially middle and high school teachers, avoid using manipulatives for reasons that include: (a) fear that students will misbehave, (b) dearth of knowledge and skill on how to teach using manipulatives, (c) avoidance of the extra time involved in the management of manipulatives, (d) believing that the use of manipulatives makes instruction less rigorous, or (e) believing that the use of manipulatives belongs in the previous grade segment. However, as students are denied opportunities to interact with the isomorphic structures that help them make meaning of abstract mathematical ideas, they experience mathematics as a series of disconnected rules and procedures that are to be memorized, and are therefore denied the opportunity to understand why a mathematical idea is important or how several procedural competencies can be applied to different contexts. As the nation prepares to embark in the Common Core State Standards (CCSS) era, with more rigorous expectations for student achievement "…educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural fluency, and application." (Alberti, 2012, pp. 25-6).

The National Research Council (2001) described conceptual understanding – or an integrated and functional grasp of mathematical ideas (p. 116) – as one of the five interwoven and interdependent strands of mathematical proficiency needed for anyone to learn mathematics successfully (p. 115-6). These five strands – conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition – lay the foundation for the CCSS' Standards of Mathematical Practice which collectively describe the behaviors of mathematically proficient students.

Moreover, to the extent that conceptual understanding plays a pivotal role in the development of procedural proficiency (National Research Council, 2001; Seigler, 2003), and that both play a significant role in students' ability to apply their mathematical knowledge to problem situations, it becomes quite clear that students who are unable to develop deep understanding of the various mathematical ideas they study will be more likely to fail in mathematics and in meeting the expectations set forth by the CCSS for content and practice. Therefore, the use of concrete manipulative materials in the instruction of secondary school mathematics plays an important role in promoting conceptual understanding that will allow students to succeed in the CCSS era.

In order to ensure that students accrue the benefits identified by research, teachers must develop proficiency in the use of manipulatives for instruction. For starters, teachers should select manipulatives that have the same isomorphic properties as the concept to be developed and that is appropriate to students' developmental level. The concept should be presented contextually as a problem-solving opportunity to be solved with the concrete manipulative(s). It is important to recall that the interaction with concrete manipulative materials by itself does not promote conceptual understanding. Teachers need to engage students in making meaning of the problem-solving experience by: a) connecting to the underlying math concept; b) incorporating a variety of manipulatives for concept exploration; c) providing verbal explanations and questioning with demonstrations; d) providing opportunities for students to interact with one another and communicate their reasoning and critique the reasoning of others as well as making meaningful connections with the concept; and e) monitoring student progress and readiness to transition from concrete to symbolic representations (Van de Walle, Karp, & Bay-Williams, 2011; Maccini & Gagnon, 2000; Anstrom, 2006).

Bruner (1966) postulated that learning goes through three stages and that all learning begins with an action. On the *enactive* or concrete stage, learning happens through touching, feeling, and manipulating. On the *iconic* or pictorial stage, learning changes to depend on visual representations that are used to summarize and represent concrete situations. On the *symbolic* or

C

- **Concrete** or ***enactive***
- **Object Manipulation**
- **Contextual Problem Solving**
- **Concept exploration in purposeful activity**
- **Students demonstrate understanding by physically manipulating objects**

R

- **Representational** or ***iconic***
- **Pictorial record of the concrete manipulations**
- **Graphic representations**
- **Students demonstrate how they are able to visualize and communicate concepts**

A

- **Abstract** or ***symbolic***
- **Use of mathematical symbols and notation to express concepts**
- **Students demonstrate understnading by using the language of mathematics**

abstract stage, words and symbols representing information do not necessarily have any inherent connections with the information being represented. The use of symbols allows students to organize information in their minds by connecting concepts and experiences together and by manipulating these symbols to a level that exceeds some of the physical limitations of the original information. Additionally, Piaget (1957, 1969) contributed to the development of constructivism by postulating that people learn best when they can experiment and invent their own generalizations without being told how to think by the teacher as they go through developmental stages in their lives - with each stage contingent upon the completion of the previous stage. Similarly, Vygotsky (1962) defined the zone of proximal development as the state between what a student already knows and understands and what she is capable of comprehending through conversations with another person. While Bruner's theory has been credited with the expansion of the use of manipulatives, the combination of these theories provides the theoretical underpinnings of the Concrete-Representations-Abstract (CRA) instructional approach (Souza, 2007).

The CRA as an Approach to Teaching with Manipulatives

The CRA instructional approach is a three-part instructional strategy where each part builds on the previous instructional instance to promote student learning and retention, and addresses conceptual knowledge by supporting the understanding of underlying concepts before the memorization of procedural "rules" (Anstrom, 2006). In the first stage students learn through the manipulation of concrete objects. It is followed by the pictorial stage, where students learn through graphic or symbolic representations of the concrete manipulations that took place in the first stage. Finally, students' progress to the abstract stage, where they learn through the use of abstract notations such as numerical and other mathematical symbols.

As a multi-sensory approach, the CRA "relies on not only visual and auditory interactions with content, but also kinesthetic and tactile interactions, through the use of hands-on manipulations of objects and matching of pictorial drawings" (Witzel, 2005, p. 50); which makes it appealing to students with diverse learning style preferences. The CRA is also considered beneficial because by holding the objects in their hands and working with them, students build mental models that represent the reality they physically manipulate (Steedly, Dragoo, Arafeh, & Luke, 2008). Thus, by implementing the CRA teachers provide an opportunity for students to use concrete manipulative materials to develop understanding of the mathematical concept under the enactive phase; foster connections between the concept and the operational procedures employed to make sense of the mathematics in the iconic phase; and make generalizations as students adopt the abstract language of mathematics to express their thinking in the symbolic phase. The effective implementation of the CRA approach has been identified as an effective strategy to support not only students with learning disabilities (Witzel, Mercer, & Miller, 2003; Steedly, et. al., 2008; Anstrom, 2006; The Access Center, 2004), but also general education students (Hughes & Riccomini, 2011; Strozier, 2012). These characteristics of the CRA are consistent with the NCTM recommendations that students explore mathematics through hands-on means as they build problem-solving and higher-order thinking skills (NCTM, 2000).

The implementation of the CRA, however, is more than just students interacting with manipulatives. "Even if children begin to make connections between manipulative and nascent ideas, physical actions with certain manipulatives may suggest different mental actions than those we wish students to learn" (Clements, 1999, p.47). Teachers should monitor student progress to help them discover and focus on the mathematics students should learn.

> One of the reasons that we as adults may overstate the power of concrete representations to deliver accurate mathematical messages is that we are "seeing" concepts that we already understand. That is, we who already have the conventional mathematical understandings can "see" correct ideas in the mathematical representations. But for children who do not have the same mathematical understandings that we have, other things can reasonably be "seen" (Ball, 1992, p. 17).

By observing and questioning students as they develop and communicate their solutions, teachers are able to ensure that students are not developing misconceptions and are building a level of conceptual understanding that is generalizable and transferable to other problem situations. Teacher planning may also scaffold the transition of students to the representational or the abstract phases of the CRA. Research suggests that by integrating the phases of the CRA approach; where the concrete and abstract phases are connected during instruction, teachers promote better understanding and transfer of the concept to novel problem situations more effectively – especially related to work with secondary students (Pashler et. Al., 2007) in either general or special education settings (Strickland & Mancini, 2013).

Another important consideration for teachers relates to student motivation. The way teachers implement the use of manipulatives in their classes may foster self-determination and intrinsic motivation – a desirable outcome of mathematics instruction. "Teachers can foster students' intrinsic motivation when they use manipulatives in ways that support students' autonomy, develop their competence, and allow them to experience relatedness with the teacher and other students" (Jones, Uribe-Florez, & Wilkins, 2001, p. 224). Additionally, the conditions that support intrinsic motivation in students seem to correlate with those identified by Marzano (2007) as the basic elements of cooperative learning which in turn points out the possibility of implementing the CRA as either a one-on-one or as a small group strategy.

The CRA and the Common Core

The CCSS calls for middle school students to "do hands on learning in geometry, algebra, and probability and statistics after building strong foundation in K-5" (CCSS, 2010). The strong foundation in K-5 is couched in specific structures such as the area model and the number line. These structures, in turn, can be leveraged to further conceptual understanding of secondary school mathematics. As students engage in the study of algebra, for example, teachers can rely on the area model and algebra tiles to deepen student understanding and procedural proficiency of binomial multiplication, quadratic trinomial factorization, as well as with the multiple representations of quadratic functions. As students look for and make use of the structures of algebraic expressions that are initially developed with manipulatives, they will be well served to deal with more complex mathematical representations.

Likewise, the Common Core makes explicit connections between content and practice standards and fluency expectations (PARCC, 2012). For instance, fluency in the operations with integers anchors operations with polynomials, and the remainder and factor theorems. By using the CRA to structure the use of two-color chips to introduce the addition and subtraction of integers (Flores, 2008), teachers pave the way to the mastery of advanced algebra. Similarly, algebra tiles, Algeblocks ®, and Hands-on Equations ® are other materials that could be used to advance the development of other mathematical concepts such as the solution of equations and the properties of equality.

The market place is filled with different manipulative materials to promote the acquisition of mathematical concepts. The CRA provides an instructional approach that allows teachers to effectively develop conceptual understanding of secondary school mathematics by grounding their procedural fluency in meaningful and connected concrete experiences. The judicious use of the CRA in secondary school mathematics paves a pathway to the successful implementation of the CCSS content and practice standards.

References

Alberti, S. (December 2012/January 2013). Common Core: Now what? *Educational Leadership, 70*(4), 24-27.

Anstrom, T. (2006). Supporting students in mathematics through the use of manipulatives. Retrieved from Center of Implementing Technology in Education: http//www.cited.org/library/resourcedocs/Supporting%20Students%20in%20Manipulatives.pdf.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. American educator: *The Professional Journal of the American Federation of Teachers, 16*(2), 14-18.

Brahier, D. J. (2009). Teaching secondary and middle school mathematics, 3rd ed. Boston: Pearson.

Bruner, J. S. (1966). Toward a theory of instruction. New York: W. W. Norton.

Burns, M. (2007). About teaching mathematics: A K – 8 resource. Sausalito, CA: Math Solutions Publications.

Clements, D. H. (1999). 'Concrete' manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45-60.

Flores, A. (2008). Subtraction of positive and negative numbers: The difference and completion approaches with chips. *Mathematics Teaching in the Middle School, 14*, 21-23.

Frei, S. (2008). Teaching Mathematics Today. Professional Development for Successful Classrooms. Huntington Beach, CA: Shell Education.

Hughes, E. M. & Riccomini, P. J. (Fall 2011). Mathematics motivation and self-efficacy of middle school students. Focus on Middle School, 24(1). Olney, MD: Association for

Childhood Education International.

Jones, B. D., L. J. Uribe-Florez, & J. L. M. Wilkins (2001). Motivating mathematics students with manipulatives: using self-determination theory to intrinsically motivate students. In Brahier, D. J. (Ed.) Motivation and Disposition: Pathways to learning Mathematics . Seventy-third Yearbook. Reston, VA: National Council of Teachers of Mathematics.

Macinni, P. & Gagnon, J. A. (2000, January). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children, 32*(5), 11.

Marzano, R. J. (2007). The art and science of teaching: A comprehensive framework for effective instruction. Alexandria, VA: ASCD.

National Council of Teachers of Mathematics, NCTM (2000). Principles and standards for school mathematics. Reston, VA: Author.

Partnership for Assessment of Readiness for College and Career, PARCC (2012). PARCC Model Content Frameworks: Mathematics Grades 3 – 12. Version 3.0.

Pashler, H., Bain, P. M., Bottge, B., Graesser, A., Koedinger, K., McDaniel, M., & Metcalfe, J. (2007). Organizing instruction and study to improve student learning. Washington, DC: U.S. Department of Education.

Piaget, J. (1957). Construction of reality in the child. London: Routledge & Kegan Paul.

Piaget, J. (1969). The mechanisms of perception. New York: Basic Books.

Post, T. (1981). The Role of Manipulative Materials in the Learning of Mathematical Concepts. In Selected Issues in Mathematics Education (pp. 109-131). Berkeley, CA: National Society for the Study of Education and National Council of Teachers of Mathematics, McCutchan Publishing Corporation.

Siegler, R. S. (2003). Implications of cognitive science research for mathematics education. In Kilpatrick, J., Martin, W. B., & Schifter, D. E. (Eds.), A research companion to principles and standards for school mathematics (pp. 219-233). Reston, VA: National Council of Teachers of Mathematics.

Steedly, K., Dragoo, K., Arafeh, S. & Luke, S. D. (2008). Effective mathematics instruction. Evidence for Education, 3(1). National Dissemination Center for Children with Disability.

Strickland, T. K. & Mancini, P. (2013). Effects of Concrete-Representational-Abstract integration on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial Special Education, 34*(3), 142-153.

Strozier, S. D. (2012). The effects of Concrete-Representational-Abstract sequence and a mnemonic strategy on algebra skills of students who struggle in math. Doctoral Dissertation: Auburn University

Souza, D. A. (2007). How the brain learns mathematics. Corwin Press.

Sowell, E.J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20*, 498-505.

The Access Center, (2004). Concrete-Representational-Abstract approach.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2011). Elementary and Middle School Mathematics: Teaching developmentally (8th ed.). Boston: Pearson

Vygotsky, L. S. (1962). Thought and language. E. Hanfmann & G. Vakar, Trans. Cambridge, MA: MIT Press.

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal 3*(2), 49 – 60.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning disabilities: An investigation of an explicit instruction model. *Learning Disabilities: Research & Practice, 18(*2), pp.121-131.

Maryland Council of Teachers of Mathematics

Calendar

April 2014	**Maryland Math Month.** Activities are in calendar format that provide math activities to help you and your students celebrate Maryland Math Month. https://www.marylandmath.org/events/mathmonth
May 3, 2014	**Eastern Shore Mini-Conference**
October 16, 2014	**Annual Meeting and Banquet @ TBA**
October 17, 2014	**MCTM Conference @** Baltimore Polytechnic High School, 1400 W Cold Spring Ln, Baltimore, MD 21209

See our website for more details. https://www.marylandmath.org/

Where are the Resources for Teaching Common Core Algebra 2?

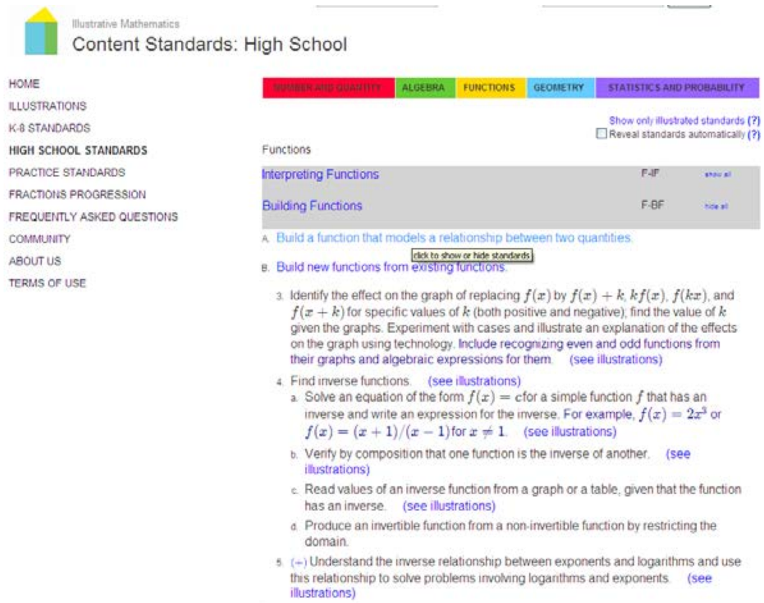
Kimberly Toms
Frederick County Public Schools

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

Algebra 2 is one of my favorite courses to teach. Helping students to understand the abstract concepts is truly rewarding. But just when I have perfected my lessons and teaching strategies, here comes the Common Core Curriculum. So what runs through my mind? What will be different? What will be the same? And most importantly, what resources are out there to help us teach this new Algebra 2 course?

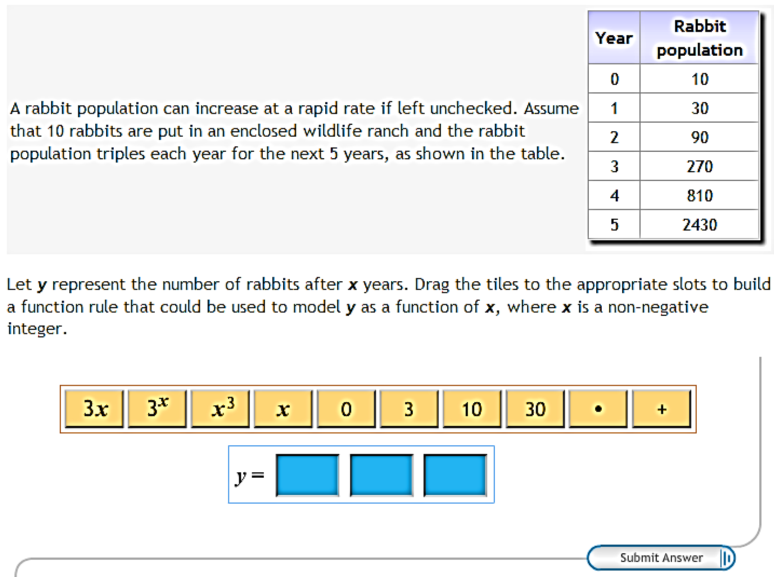
There are a number of challenges resulting in Maryland's adoption of the Common Core State Standards. One of the biggest challenges is the realignment of content standards. Teachers must review courses carefully to get a sense of the vertical development of concepts. Topics traditionally reserved for Algebra 2, like radical and rational functions, can now be found in Common Core Algebra 1 course. Common Cores Algebra 2 will include the new topics: descriptive statistics (inferences and conclusions from data), applications of probability (rules of probability and conditional probability) and trigonometry (understanding radian measurement as arc length, using the unit circle to extend trigonometry functions beyond quadrant one, choosing trigonometric functions to model periodic behavior and solving trigonometric equations using identities). Along with a realignment of content, teachers are also challenged to concurrently develop the set of critical process standards called the Mathematical Practices and prepare students for the PARCC assessment. This article provides a description of the current online resources to help teachers with the transition to Common Core Algebra 2.

Illustrative Mathematics (www.illustrativemathematics.org) "provides guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the Common Core State Standards, and by publishing other tools that support implementation of the standards" (Illustrative Mathematics, 2013). This website is an initiative of the Institute of Mathematics and Education and funded by the Bill and Melinda Gates Foundation. It is a work in progress, creating a selection of mathematical tasks for each of the kindergarten through high school Common Core State Standards. The goal is to have a variety of mathematical tasks per standard that focuses on a standard and shows the connections with other standards, and to include teaching and assessment tasks over a broad range of levels. The tasks on this website are written by teachers, educators, and mathematicians. The content on this site is licensed under an agreement which gives teachers permission to copy, distribute, and adapt the work as long as they attribute the work for noncommercial use with the intent to share their adapted work. The site is user-friendly, and the tasks are easy to find by conceptual category, cluster and standard (A.REI.11). There are 178 mathematical tasks available on this website that correlate to the standards in the Algebra 2 Common Core Curriculum. Educators who register on the site can comment on and rate tasks and lessons and submit tasks for vetting and possible publication.



The **Mathematics Common Core Toolbox** (http://www.ccsstoolbox.org) was “created through a collaboration of the Charles A. Dana Center at the University of Texas at Austin and Agile Mind with partial funding from the Bill & Melinda Gates Foundation” (Mathematics Common Core Toolbox, 2012). It is a resource to help school districts better understand and implement the Common Core State Mathematics Standards. This resource has several main areas: 1) the Standards for Mathematical Practice with links to Dana Center Assessments and MARS (Mathematics Assessment Research Service) tasks; 2) the Standards for Math Content, which includes the Common Core document and ideas for developing concepts (functions, volume, rate and proportionality) across grades; 3) resources for implementation, which includes the PARCC prototyping project that contains 5 innovative high school assessment tasks that reflect the direction of the PARCC end-of-year course test; and 4) other helpful information with links to websites that also have CCSS resources. Here is an example of an innovative assessment task called Rabbit Populations. For part a, students write an explicit expression for a sequence from a table. The student will drag the tile into the appropriate slot to build the equation. For part b, students demonstrate their understanding of the meaning of specific equation parameters in the context of the growth of a rabbit population. The students have to choose which statements must be true for the model. They have to select all that apply. Math questions are not written this way in a typcial math class. Teachers need to familiarize themselves with the assessment tasks that will be on PARCC and start preparing their students for them.

Part a

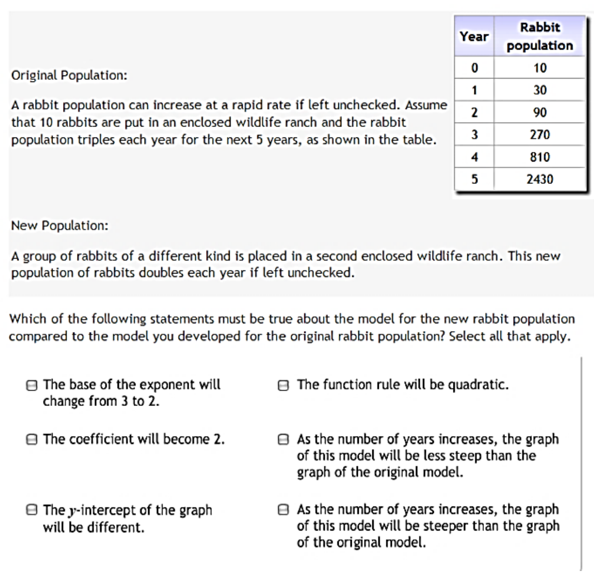


A rabbit population can increase at a rapid rate if left unchecked. Assume that 10 rabbits are put in an enclosed wildlife ranch and the rabbit population triples each year for the next 5 years, as shown in the table.

Year	Rabbit population
0	10
1	30
2	90
3	270
4	810
5	2430

Let y represent the number of rabbits after x years. Drag the tiles to the appropriate slots to build a function rule that could be used to model y as a function of x, where x is a non-negative integer.

Part b



Original Population:

A rabbit population can increase at a rapid rate if left unchecked. Assume that 10 rabbits are put in an enclosed wildlife ranch and the rabbit population triples each year for the next 5 years, as shown in the table.

Year	Rabbit population
0	10
1	30
2	90
3	270
4	810
5	2430

New Population:

A group of rabbits of a different kind is placed in a second enclosed wildlife ranch. This new population of rabbits doubles each year if left unchecked.

Which of the following statements must be true about the model for the new rabbit population compared to the model you developed for the original rabbit population? Select all that apply.

- The base of the exponent will change from 3 to 2.
- The function rule will be quadratic.
- The coefficient will become 2.
- As the number of years increases, the graph of this model will be less steep than the graph of the original model.
- The y-intercept of the graph will be different.
- As the number of years increases, the graph of this model will be steeper than the graph of the original model.

Tools for the Common Core Standards

Dr. William McCallum, head of the mathematics Department at the University of Arizona and one of the lead authors of the Common Core State Standards for Mathematics, blogs about updates and reports on projects that support the implemantation of the Common Core State Standards for Mathematics on the website http://commoncoretools.me/author/wgmccallum. This resource has the following tools for teachers: Illustrative Math Project, progression documents for the common core and K – 8 standards by domains. There is a link that provides information about Illustrative Mathematics with a link to the website and information on how teachers can write and submit mathematical tasks to be considered for publishing. Lastly, there is a group of 17 different forums (with 192 topics and 467 replies) where educators can ask questions and give input. Registration for the site is free, and registered users will be notified through email of new posts.

Members of our own Maryland Council of Teachers of Mathematics have developed the Core Challenge website (http://www.corechallenge.org/), offering workshops and technological resources for teachers. Check it out for pencasts on difficult teaching topics and dates for special events. And the MCTM website (www.marylandmath.org) continues to highlight resources for all teachers as we face the transition to the Common Core.

These are challenging times for teachers of mathematics at all grade levels. It is also an exciting time. By raising standards for our students through the Mathematics Common Core State Standards and the use of the Mathematical Practices students will be better prepared for what is expected of them in college and in the work force. Many mathematics educators and non-profit organizations are hard at work developing resources for teachers to use in this transition to the new Common Core Standards. With them, we can face teaching Algebra 2 – and other math courses – with confidence.

References

Institute for Mathematics Education. *Illustrative Mathematics*. Retrieved March 22, 2013, from www.illustrativemathematics.org/

Maryland Council of Teachers of Mathematics, *Resources* (https://www.marylandmath.org/resources) and *Professional Resources* (https://www.marylandmath.org/news/proresources). Retrieved June 10, 2013.

McCallum, D. W. (n.d.). Bill McCallum | Tools for the Common Core Standards. *Tools for the Common Core Standards | News about tools that are being developed to support implementation of the Common Core State Standards*. Retrieved April 1, 2013, from http://commoncoretools.me/author/wgmccallum/

The Charles A. Dana Center at the University of Texas at Austin. *The Mathematics Common Core Toolbox | Network Solutions*. Retrieved March 15, 2013, from http://www.ccsstoolbox.org/

Insight for New Teachers: The Importance of Review

Brittany Beresford
Frederick County Public School

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

How many times a week (day, hour) does someone ask you a question that you cannot remember the answer to? For me, it happens fairly often. Some examples: "Who is the author of your favorite book?" (Khaled Hosseini, who wrote *A Thousand Splendid Suns* – I had to Google it). "When is Leah's birthday?" (October 23 – thank you, Facebook). And finally, "How do you solve this Calculus problem?" (I still do not have the answer to this – I got frustrated and sent the student to another teacher). But I love Calculus! I have loved every math class that I have taken since Algebra 1 in eighth grade. Yet I could not remember the steps for implicit differentiation. My point is not to draw attention to my personal embarrassments but to lead to a discussion about what we expect our students to remember.

We have a difficult time remembering things we have an interest in and an even more difficult time remembering things we have little interest in. Many students have little interest in Algebra. But we expect them to learn, understand, and remember new material almost daily, then go back at the end of the year and remember everything for the final exam.

The purpose of this article is not to give some magical insight into how to get our students to love and remember everything we teach them. I am only in my second year of teaching, so I still have a long way to go until I can confidently give such teaching insight. However, I have learned a lot from my first year as a teacher that has led me to implement several changes and improvements in my second year. Hopefully this article will help new teachers to avoid some of the errors that I made and even give veteran teachers some new perspective.

In my first year, I gave my Algebra 1 classes a quiz on solving equations. I was shocked by how many conceptual errors my students made. I knew that I had taught everything that they needed to know for the quiz, and no one had asked me any questions beforehand. Silly me. I quickly learned that I need to do a better job of consistently reviewing what I teach, making connections between concepts, and checking for student understanding. This article begins with why it is essential to review and then gives methods for implementing that review in the classroom.

Furthermore, with the recent implementation of the Common Core curriculum, it is still important to review what we teach to our students. Although the goal of Common Core is to instill in our students the ability to be life-long learners and to prepare them for success in the workplace, we still need to make sure that they have the foundational skills necessary to solve such problems. I have found that my students in a Common Core Algebra class are able to use appropriate tools strategically and persevere in solving problems, like the Standards of Mathematical Practice state (Common Core State Standards Initiative, 2010). However, they still make errors in solving equations or simplifying expressions. The following ideas will help with these skills while still following and covering the Common Core curriculum.

Staying on task

In order to effectively teach students, it is important to understand how they retain and recall information. During long lectures, I find myself zoning out and having a difficult time staying focused on the speaker. Why should I expect my students to pay better attention than I can? Although I would like to believe that every word that I say fascinates my students - in reality, this is far from the truth. Therefore, we as teachers must present the material in ways that engage our students. But developing creative, student-centered lessons every day is impossible, especially for new teachers. Instead of stressing over creating imaginative lessons, I learned to focus on the classroom routine. I like to present the material and then have students practice in class in order to ensure they gained a solid understanding. (The “flipped classroom” emphasizes this practice in class. Because I have block scheduling, I have enough time to do the lesson and have students practice. But for a schedule with shorter classes or even certain classes in block scheduling, this might be a practice you would want to look into.)

“Students can be expected, under the best of circumstances, to be on-task about ¾ of the time they are receiving classroom instruction” (Banikowski, 1999, p. 7). So if you teach for twenty minutes - in the best possible situation - your students were not paying attention for five of those minutes. A lot can be lost in those five minutes. How can teachers ensure that students have the opportunity to later gain that lost information?

Even if students focus the entire class, it is one thing to understand a math problem when the teacher guides you through it and quite another to understand it independently hours later at home. Schwartz (1999) states, "Even students who have a good grasp of math and have been taught effectively can struggle as they attempt to do homework eight to 24 hours after a class presentation." We need to ensure that students understand the homework before they leave the classroom. Students become frustrated when they follow along with the teacher in class, go home and cannot complete the problems on their own, and then arrive at class the next day only to move on to new material. Nancy Frey and Douglas Fisher (2011) discuss how homework should only be given to provide more practice after the content has been fully covered in class and students already have had an opportunity to practice and receive feedback. I notice a much greater amount of on-task students when I have had the opportunity to circulate and give feedback because students are more confident in their ability. How can we set up a classroom routine that provides for this in-class practice and feedback?

Review and feedback

I feel that the best way to help students to succeed in Algebra is to consistently review old concepts and to always connect those old concepts to new information. Furthermore, I learned in my first year to always check for students' understanding using multiple formative assessments in order to ensure that they are completing the work correctly.

Schwartz (1999) also states, "Going over the basics helps the students to see learning math as a 'building block' process, which must be approached in increments." Concepts in math build on one another, so we cannot move on without students understanding the foundation. Because of this, we must reinforce those beginning topics whenever we can. I always feel that I understand concepts better the second time I encounter them, and I see that with my students as well.

Students often forget how to solve absolute value equations. This year, I made sure to explain why we make two equations. I did this by using an example like $|x-3|=6$ and explaining that we are taking the absolute value of "something" and getting an answer of 6. Then I asked what we could be taking the absolute value of, and my students remembered that both the absolute value of 6 and -6 would produce an answer of 6. Therefore, x – 3 could equal 6 or -6. This helped them to understand absolute value equations the first time we covered them, and then when they came up again all I had to ask was, “How many solutions should we get?” and “Why do we make two equations to get those two solutions?”

We need to review in order to get students to become comfortable with the math, but feedback is just as important in developing successful math students. Konald, Miller, and Konald (2004) discuss the importance of feedback and state, “Feedback is an important aspect of every school day and plays a critical role in the teaching/learning process,” and according to Miller, “The primary purposes for providing feedback are to reinforce appropriate learner behavior, let students know how they are doing, and extend learning opportunities” (as cited in Konald et al., 2004, para. 5). In my previous example of my students doing poorly on the solving equations quiz, I probably could have avoided such bad scores if I had done a better job of checking for understanding and providing appropriate feedback.

After realizing how I could improve my teaching from last year, I made some changes and additions to my classroom routine this year. I would like to discuss four specific ways that I review and provide feedback – chunking the lesson, Exit Tickets, Warm-Ups, and Unit Reviews.

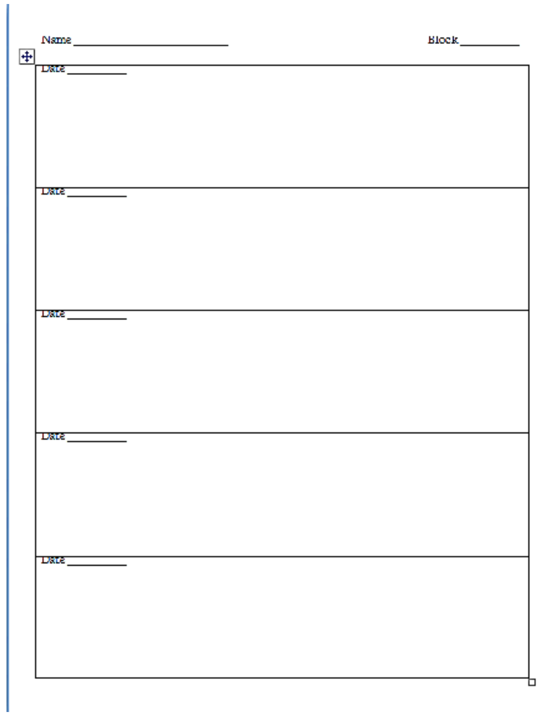
Chunking the lesson

In order to combat the issues of students staying focused at most ¾ of the time and them going home confused by their homework, I began the practice of “chunking” my lesson. Another teacher at my school recommended this in order to keep students focused and provide beneficial classwork time. Also, Banikowski (1999) recommends, “Using contrast can assist in arousing attention or refocusing attention” (p. 7). Chunking provides for this contrast.

Chunking means alternating between notes and independent or group work in order for the students to practice what they just learned in the class examples. Last year, I would do all of the notes at once, and I could visibly see students losing focus. By chunking the lesson, I only teach one or two new things at a time, and then immediately give them time to practice (for about 10-15 minutes each). In this way, they have the opportunity to ask me questions, and I can walk around and check their work before moving on to something new. Since making this change from last year, I have noticed an increase in the number of on-task students and a decrease in the number of homework questions the next day. Furthermore, there are many tasks that align with the Common Core Standards that engage students. When implemented correctly, these tasks keep students on-task, show them connections to math outside the classroom, and help them to develop important skills and practices.

Exit Tickets

Although I call on students during class and circulate during classwork, I also like to have my students complete Exit Tickets in order to check their understanding. As seen in Figure 1, I give students a sheet to complete their Exit Tickets on every two weeks (five blocks on the front and five on the back). At the end of class, I put a problem on the board for the students to complete on their sheet. I collect these sheets when they are done and check them daily. If the solution is correct, I give them a check. If the solution is incorrect, I circle their errors or write down hints to help them. I give the sheets back the next day. If they did not get the problem correct, they can attempt it again, asking me questions if necessary. They have until the last day of the two weeks to correct their mistakes, and then the Exit Tickets are graded (two points for each problem they have a check on).


Name ____________ Block ______

Date ______

Date ______

Date ______

Date ______

Date ______

Figure 1

This is probably my favorite form of assessment and feedback. It only takes up a few minutes at the end of the lesson, and I can quickly check the answers after class. A lot of the times the students make the same errors, which I can address at the beginning of class the next day. The Exit Tickets help me to correct students' mistakes before they must demonstrate knowledge on a quiz or a test.

Warm-Ups

To review the methods I have discussed so far, I have students immediately practice in class, give Exit Tickets at the end of the lesson, and have students practice after school with homework. These are three separate ways for the students to show me that they have learned the concepts from that day. Then they must complete a Warm-Up as soon as they come into class the next day. This is to show that they still remember the concepts from the day before and to connect the previous lesson to the new lesson.

Wong and Wong (2009) say, "Your first priority when the class starts is to get the students to work" (p. 123). When you have a Warm-Up posted, it shows the students that you are organized and ready to work that day, and you expect your students to get in that mindset as well. Warm-Ups also help to clarify any questions from the day before and allow you to re-emphasize important points.

I actually use the same form for Warm-Ups as I do for the Exit Tickets. However, students hold onto the Warm-Ups, and I collect them after the two weeks. These I only grade for completion. Warm-Ups are a great way to review from the previous day and to lead in to the new lesson.

Unit Reviews

Frey and Fisher (2011) also state, "Unfortunately, in too many cases, students never return to previous content during the school year and thus schools are doomed to spend weeks reviewing before high-stakes accountability assessments." In order to avoid this, they suggest spiral reviews to practice familiar concepts throughout the year.

I try to implement these spiral reviews by giving my students what I call Unit Reviews every week. (Another teacher had also shown me this idea – new teachers, take advantage of all of your resources!) Unit Reviews contain ten problems taken from everything we have learned so far that year (see Figure 2). Students have one week to complete these. However, they can turn their Reviews in as often as they like, and I will check off the ones that are correct and hand them back. In this way, students learn from their mistakes and have the opportunity to get the correct answers. The Unit Reviews have definitely been beneficial in my students remembering topics that we have covered since the beginning of the year.

Conclusion

I learned many things during my first year of teaching. Do not try and do everything on your own – ask for help from veteran teachers. Take time for yourself. Caffeine is your new best friend.

But the one thing that I learned that greatly influenced my teaching this year is the importance of review and feedback. Students will remember concepts until the test, and then it is like they open a back window in their head and let all that they learned fly out and disappear. Our job as teachers is to keep those learned ideas in our students' minds, where they can easily recall them.

Chunking the lesson gives the students immediate practice so that they can begin learning the correct way. Exit Tickets show that they did learn the objectives and provides a way to give daily feedback. Warm-Ups help the students recall the information that they learned the previous day and to connect that information to the new lesson. Finally, Unit Reviews force students to recall information from previous topics.

In order to help students to succeed, we should also make sure that they know the “why” of what we are doing and not just the “how,” like I tried to do for solving absolute value equations. Students can normally better recall information learned this way.

Review and feedback will help the class run more smoothly because students will feel confident in the math that they are performing and will be ready to acquire new information.

Algebra 1 Unit Review #10

Name ______________________ Due Date ______________ *No Work, No Credit*

1.) Simplify $15 \div 3 \cdot 5 - 4^2$

2.) Simplify $8(3x - 7y)$

3.) If $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ -1 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & -1 \\ -1 & -2 & 0 \end{bmatrix}$, find A + B.

4.) Solve $\frac{4x+5}{7} = 7$

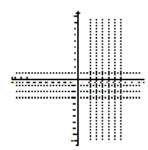
5.) Solve $xy - b = c$, for y.

Algebra 1 Unit Review #10

6.) Solve $6 + |x + 1| = 9$

7.) Solve and graph the inequality $-3(k - 2) > 12$

8.) Sketch the graph. $4x + 2y = 10$



9.) Calculate the slope of the line that goes through the points $(4,6), (-2,-8)$

10.) Rewrite in slope-intercept form: $3x + 4y = 12$.

Figure 2

References

Banikowski, A. K. (1999). Strategies to Enhance Memory Based on Brain-Research. *Focus on exceptional children*. Retrieved from http://sc-boces.org/english/IMC/Focus/Memory_strategies2.pdf.

Common Core State Standards Initiative (CCSSI). (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

Frey, N., & Fisher, D. (2011). High-Quality Homework. *Principal Leadership 12* (2), 56-58.

Konold, K.E., Miller, S,P., & Konold, K.B. (2004). Using Teacher Feedback to Enhance Student Learning. *Teaching Exceptional Children 36*(6), 64-69.

Schwartz, A.E. (1999). CURRICULUM: Overcoming Fear and Succeeding in Math; Lessons from a Distinguished Teacher at Mercer. ***The Hispanic outlook in higher education*. Retrieved from** http://taurus.hood.edu:2727/cv_544896/docview/219205489/1398456656948D935CC/2?accountid=11467.

Wong, H.K, & Wong, R.T. (2009). *The first days of school: How to be an effective teacher*. Mountain View, CA: Harry K. Wong Publications.

Puzzler

Answer: *The problem you describe is to draw a bipartite graph of 3 nodes connected in all ways to 3 nodes, all embedded in the plane. The graph is called K3,3. A famous theorem of Kuratowsky says that all graphs can be embedded in the plane, EXCEPT those containing a subgraph that is topologically equivalent to K3,3 or K5 (the complete graph on 5 vertices, i.e., the graph with 5 nodes and 10 edges). So your problem is a minimal example of a graph that cannot be embedded in the plane.*

*The proofs that K5 and K3,3 are non-planar are really quite easy, and only depend on Euler's Theorem that F-E+V=2 for a planar graph. For K3,3 V is 6 and E is 9, so F would have to be 5. But each face has at least 4 edges, so E >= (F*4)/2 = 10, contradiction. For K5 V is 5 and E is 10, so F = 7. In this case each face has at least 3 edges, so E >= (F*3)/2 = 10.5, contradiction.*

The difficult part of Kuratowsky is the proof in the other direction!

*A quick, informal proof by contradiction without assuming Euler's Theorem: Using a map in which the houses are 1, 2, and 3 and the utilities are A, B, and C, there must be continuous lines that connect the buildings and divide the area into three sections bounded by the loops A-1-B-2-A, A-1-B-3-A, and A-2-B-3-A. (One of the areas is the infinite plane *around* whichever loop is the outer edge of the network.) C must be in one of these three areas; whichever area it is in, either 1, or 2, or 3, is *not* part of the loop that rings its area and hence is inaccessible to C.*

*The usual quibble is to solve the puzzle by running one of the pipes underneath one of houses on its way to another house; the puzzle's instructions forbid crossing other *pipes*, but not crossing other *houses*.*

From: http://mathforum.org/rec_puzzles_archive/geometry.html

Professional Development of Middle School Mathematics Teachers

Melissa Biondo Hupe
Thurmont Middle School
Frederick County Public Schools

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College

There has been a recent focus, in published research and recent reports, on the mathematical education of teachers. An interesting topic that is not talked about as often is the education, or professional development, of those who are already teachers. It is more important than ever, in this era of the Common Core, for middle school teachers to continue their education through appropriate professional development.

Current priorities and practices in education have been shaped by traditions of school mathematics that are mostly outdated. An organization that is a leader in this area is The Conference Board of the Mathematical Sciences (CBMS), an organization consisting of sixteen professional societies sharing a common primary objective to increase knowledge in one or more of the mathematical sciences. The purpose of CBMS is to promote understanding and cooperation among these national organizations so that they work together and support each other in their efforts to promote research, improve education, and expand the uses of mathematics. In 2001, CBMS published a report known as *The Mathematical Education of Teachers (MET I)*. This report described current thinking on curriculum and policy issues affecting the mathematical education of K-12 educators. The authors of the MET I document had hoped to discuss the professional development of teacher; however, due to an abundance of information on initial preparation of teachers, the report was unable to cover this important topic.

Since the publication of the MET I the creation and implementation of the Common Core State Standards (CCSS) has come about. The Standards (Common Core State Standards Initiative, 2010) are changing the way we teach students, and therefore a change is needed in how we keep our teachers up to date. It became apparent that the MET I needed to be updated. In 2012, a second report known as *The Mathematical Education of Teachers II (MET II)* (CBMS, 2012) was published. Finally, the report reaffirmed that mathematical knowledge for teaching can and should grow throughout a teacher's career: the report has now addressed the professional development of teachers of mathematics.

MET II and Middle Level Math Teacher Professional Development

For current teachers in the middle grades, professional development is going to be extremely important. Much of the middle level mathematics in the CCSS will be topics that current middle level teachers have never taught before. This means that professional development should focus on helping teachers understand these topics that they may be teaching at a deep level. The MET II makes several suggestions of how this can be done. For example, teachers may choose to work together in professional learning communities where they can discuss a specific topic for some time; they can study how related topics progress across grade levels; or they may choose to watch a lesson taught by a curriculum specialist and then discuss lesson planning or plan additional lessons. Teachers may also choose to take courses in specific middle level teacher programs. They can share students' work and ideas of how they think about mathematics. Assessment results can be shared and, depending on the outcome, appropriate tasks can then be designed. Also, teachers can work with mathematicians or professionals from math-intensive professions, allowing them to make meaningful connections (CBMS, 2012, pp. 45-50).

The MET II report also describes other professional development ideas for high school level teachers; however they could be just as useful at the middle school level. This first idea is known as *math teachers' circles*. The national Math Teachers' Circle Network (http://theteacherscircle.org/) is a program developed by the American Institute of Mathematics. This program's mission is to establish the foundation for a culture of problem solving among middle school math teachers in the U.S. By fostering the confidence to tackle open-ended math problems, middle school teachers become better equipped to initiate more student-centered, inquiry-based pedagogies in their classrooms. The program has two primary goals. The first is to engage middle school math teachers in mathematical problem solving and involve them in an ongoing dialogue about math with students, colleagues, and professional mathematicians; and the second is to provide guidance, materials, and resources to middle school math teachers that will enable them to promote open-ended problem solving as a way of learning, thinking about, and practicing mathematics in their classrooms. According to MTC, research on math teachers' circles has recently begun to provide evidence about how the program affects middle school teachers. Most of the studies reported positive results in favor of Math Teachers' Circles. For example, a study conducted at the University of Colorado, Colorado Springs indicated that after one year of Math Teachers' Circle participation, teachers reported increased feelings of being pedagogically prepared, more attunement to investigate culture and practices, and increased personal math teaching self-efficacy. Teachers also reported increased use of inquiry-based teaching practices. Additional research was done during a weeklong intensive summer Math Teachers' Circle workshop in which middle school math teachers participated. These teachers significantly increased their scores on a standard test measuring mathematical knowledge for teaching ("Math Teachers' Circle Network: Outcomes", n.d.).

A second idea is known as an *immersion experience*. According to the MET II report, teaching mathematics is greatly enhanced when teachers work themselves as mathematicians and statisticians gaining research experience. In an immersion experience, a practicing teacher works on a "small, low threshold, high ceiling cluster of ideas for a sustained period of time" (CBMS, 2012, p. 68). This experience has profession-specific benefits. Teachers have the opportunity to work on mathematics and are quickly reminded of the frustration, confusion, and struggle that are natural parts of being a learner. It helps them connect ideas that are usually seen as quite different, and it keeps alive the passion that the teacher gained in his or her undergraduate study. These programs should be designed and implemented by mathematicians or statisticians.

A third idea known as *lesson study* consists of teachers' working in small teams including fellow teachers, mathematicians, mathematics educators, and administrators. The teams carefully and collaboratively create lesson plans designed to meet both content goals and general learning or affective goals for students. Then, one or more members of the team teach the lesson while the other team members observe the lesson implementation. The team then debriefs the lesson and makes revisions, sometimes teaching the revised lesson to another group of students. Mathematicians can be helpful by considering the goals of the lesson, tasks to include, and issues to address when revising the lesson (CBMS, 2012, p. 68).

No matter how the professional development is designed, teachers should be challenging one another to think or problem solve. In any case, "the best professional development is ongoing, directly relevant to the work of teaching mathematics, and focused on mathematical ideas" (CBMS, 2012, p. 32).

Different Views on the Professional Development of Middle School Math Teachers

Dr. Christy Graybeal is an Assistant Professor of Education at Hood College in Frederick, Maryland and she is a former middle school math teacher in Montgomery County. She has been teaching both elementary methods courses and graduate level classes for six years. In addition to teaching, Dr. Graybeal is the Past-President of the Association of Maryland Mathematics Teachers Educators (AMMTE) which is an affiliate of the national organization the Association of Mathematics Teachers Educators, the largest professional organization devoted to the improvement of mathematics teacher education.

Graybeal feels the most important aspect of professional development for teachers is that, no matter what it looks like, it should encourage teachers to ask questions about mathematics, it should cause them to continue to be curious about mathematics, and it should make them want to learn. She also feels that within the school system, teachers should have a common planning time and be given opportunities to plan lessons together, observe each other teaching these lessons, and be able to have time afterwards to do an in depth analysis of each lesson. In other words, they should have an opportunity to engage in lesson study style of professional development suggested in the MET II document. Dr. Graybeal explained that the Japanese model of professional development allows for teachers to complete a lesson study approximately once a month. Over the years teachers have been able to, together, design and implement a multitude of math lessons. Dr. Graybeal is also interested in the idea of math circles; in fact she recently attended a workshop on math circles with other mathematicians and math specialists. The ultimate plan is to have two teachers from each middle school in the county get together over the summer for a one week immersion program and then continue to meet throughout the school year. Mathematicians would pose problems to teachers, problems of the real world, for them to solve. Dr. Graybeal thinks this is a wonderful idea, but states that often times mathematicians only want to discuss the math, and suggests that they spend time also discussing how the math ties into education and into the classroom. Teachers need more than just content development; they should think about the pedagogy as well (C. Graybeal, personal communication, July 8, 2013).

Another person with a valuable perspective on teacher professional development is Lisa Vaeth, who taught at Thurmont Middle School in Frederick County Maryland for 20 years before becoming the school's Math Specialist. She has been in that position for nine years, and deals directly with new teachers and teacher professional development.

Ms. Vaeth had very different ideas on professional development from those of the MET reports and Dr. Graybeal. She believes that professional development topics and how the sessions are organized really should pertain to the needs of the school system at that time and should be driven by data. She gave the following example: for the upcoming school year professional development with the middle school math teachers will need to focus on the new CCSS being introduced fully in the 7^{th} and 8^{th} grades, the unit plan designs given by the county, and then lessons for these standards and units. She feels that math circles and lesson studies would be great, but would need to happen during the teachers' professional learning community (PLC) time during the weeks when professional development was not scheduled (L. Vaeth, personal communication, July 14, 2013).

Challenges of Professional Development

Dr. Graybeal reiterates that the biggest challenge is helping teachers stay curious, and excited about math, and to get teachers to learn from their students, being interested in how the students are thinking about the mathematics. She also feels that, although challenging, it is important for teachers to be interested in what teachers are doing around the world. Finally, she feels that the biggest obstacle is to find professional development that is going to encourage teachers to ask the right questions (C. Graybeal, personal communication, July 8, 2013).

Ms. Vaeth mentioned a huge obstacle in teacher professional development that many can agree with, time. It is hard to find time for her to meet with teachers and for teachers to find the time to meet with each other. Often, grade level teaming and other school issues take priority in the teachers' time. A second challenge she mentioned was state and county mandated testing. She feels that "teachers no longer have the freedom they used to with looking at their own individual students and circumstances. The teachers do not even have the time to take advantage of 'teachable moments' because the teachers need to keep moving and get through the curriculum for testing purposes." Finally she said that testing takes the fun out of math (L. Vaeth, personal communication, July 14, 2013).

CCSSO and Professional Development

I believe that schools that have the most effective teachers and therefore the most successful students are part of school systems that place a lot of value on professional development. One organization that collected information on state policies on professional development is the Council of Chief State School Officers (CCSSO), a nonpartisan, nationwide, nonprofit organization of public officials who head departments of elementary and secondary education in the U.S. (CCSSO, 2013). In 2008, the organization found that only 24 states reported having a policy aligning professional development with state content standards, 20 states reported providing funding to schools or districts to support professional development that is aligned with state content standards, and 19 states reported enforcement of the provision of professional development aligned to standards through monitoring, evaluation, or required documentation (Stillman & Blank, 2009, p. 22). A more promising statistic is that in 2008, 50 states had a policy specifying requirements for professional development to renew teacher licenses. However, the majority of these states require six semester credit hours of professional development, approximately every five years (Stillman & Blank, 2009, p. 22). Hopefully, the number of states that align professional development with state content standards will increase as more research is done that shows the effects of professional development on gains in student achievement. The CCSSO conducted studies using an experimental design approach and reported a systematic analysis of 16 studies, 12 of which focused on mathematics. They then looked for common patterns among the successful programs. The successful program designs included a strong emphasis on teachers' learning specific subject content as well as pedagogical content for how to teach the content to the students (Blank & de las Alas, 2009, p. 27). Also, the implementation of professional development included multiple activities to provide follow-up reinforcement of learning, assistance with implementation, and support for teachers from mentors and colleagues in their schools. Finally, the programs lasted for six months or more and the average contact time with teachers in programs activities was about 90 hours (Blank & de las Alas, 2009, p. 27).

Conclusions

The CCSSO was able to scientifically determine what professional development should look like overall. It is valuable, however, to discuss specifically how professional development can reach these goals. Two suggestions mentioned in the MET II for professional development methods were math circles and lesson study. Both of these methods could lead to successful and productive professional development among middle school math teachers. Although it could be challenging to find mathematicians who have time to meet with teachers, if it were possible then math circles, could definitely be beneficial. Middle school math teachers are not often exposed to higher level mathematics and due to this their interest and curiosity in mathematical learning may begin to dwindle. Challenging middle school math teachers and providing ways for them to engage in mathematical practices could positively affect their teaching in many ways. I also agree with Dr. Graybeal that in order to get the most out of this professional development method, there must exist some conversation on how the outcomes, activities, or practices can be taken back into the classroom and used effectively. As for lesson study, if time would allow, I think this method would be most beneficial and really create effective teachers and high achieving students.

References

Blank, R. K., de las Alas, N. (2009). *CCSSO - Effects of teacher professional development gains in student achievement: How meta-analysis provides evidence useful to education leaders*. Retrieved from Council of Chief State School Officers website: http://www.ccsso.org/Resources/Publications/Effects_of_Teacher_Professional_Development_Gains_in_Student_Achievement_How_Meta_Analysis_Provides_Evidence_Useful_to_Education_Leaders_.html

Common Core State Standards Initiative (2013). Retrieved from http://www.corestandards.org/

Conference Board of the Mathematical Sciences (2001). *The mathematical education of teachers.* Washington, D.C: Mathematical Association of America, in cooperation with American Mathematical Society, Providence, R.I.

Conference Board of the Mathematical Sciences (2012). *The mathematical education of teachers II*. Washington, D.C.: Mathematical Association of America, in cooperation with American Mathematical Society, Providence, R.I.

Math Teachers' Circle Network: Outcomes. (n.d.). Retrieved from http://www.mathteacherscircle.org/about/outcomes.html

Stillman, L., & Blank, R. K. (2009). *Key state education policies on pk-12 education:2008*. Retrieved from Council of Chief State School Officers website: http://www.ccsso.org/Documents/2008/Key_State_Education_Policy

The Council of Chief State School Officers (2013). http://www.ccsso.org/Who_We_Are.html

The Maryland Council of Teachers of Mathematics (MCTM) is the professional organization for Maryland's teachers of mathematics. Our members represent all levels of mathematics educators, from preschool through college. We are an affiliate of the National Council of Teachers of Mathematics (NCTM). Our goal is to support teachers in their professional endeavors and help them to become agents of change in mathematics education.

First Name: ______________________ Last Name: _________________________

Home Address: ____

City: ________________________ State: ________ Zip: _____________________

Home Phone: _______________ E-mail address: __________________________

MCTM does not share email addresses

School System or Affiliation: ____

Level: Check applicable categories:

__ Early Childhood (Pre-Kindergarten to Grade 2)
__ Elementary (Grades 3 – 5)
__ Middle School (Grades 6 – 8)
__ High School (Grades 9 – 12)
__ Higher Education (Grade 13+)

Signature: _______________________________________ Date:____________

Join on-line at http://www.marylandmath.org or mail a $15 check, made out to MCTM, and this completed form to:

Ed Nolan
10299 Huron Trail
New Market, MD 21774

MCTM Board of Directors

The MCTM Board of Directors and Committee Chairs are here to serve you. To contact a member of the board or committee, click on the e-mail link provided.

President	Andrew Bleichfeld	andrew.bleichfeld@hcps.org
Past President	Sue Vohrer	svohrer@aacps.org
Executive Director	Ed Nolan	Edward_C_Nolan@mcpsmd.org
Recording Secretary	Ming Tomayko	mtomayko@towson.edu
Treasurer	Dolores Bonincontri	lulubon713@yahoo.com
Central Region Rep.	Sue Thompson	sthompson@bcps.org
Western Region Rep.	Scott Trexler	setrexl@carrollk12.org
Southern Region Rep.	Marcie Sun	Marcia.Frank@pgcps.org
Eastern Shore Rep.	Jeanine Brizendine	jbrizendine@gmail.com
Early Childhood Level Rep.	Mark Zimmerman	mzimmerman@bcps.org
Intermediate Level Rep.	Verna Washington	Verna_L_Washington@mcpsmd.org
Middle Level Rep.	Babette Margolies	babsmar@gmail.com
High School Level Rep.	John Fontinell	jfontinell@bcps.org
Pre-service Teacher Rep.	Kelly Mattingly	kmatti6@students.towson.edu
College Level Rep.	Christy Graybeal	graybeal@hood.edu
Member-at-Large	Luis Lima	LLima@bcps.k12.md.us
Membership Chairperson	Jenny Novak	jnovak@hcpss.org
Nominations Chairperson	Camille Tomlin	camtom47@hotmail.com
New Teacher Resources	Karen Vaden	Karen_Manns-Vaden@hcpss.org
Awards Chairperson	Christine Oberdorf	christineoberdorf@comcast.net
Math Month Chairperson	Bonnie Yost	byost@bcps.org
Mathalon Coordinator	Stewart Saphier	StewSaphier@aol.com
Webmaster	Bethany Kidd	bkidd@bcps.org
Newsletter Editor-in-Chief	Christy Graybeal	graybeal@hood.edu
Banner Editor-in-Chief	Tricia Strickland	strickland@hood.edu
Historian	Babette Margolies	babsmar@gmail.com
NCTM Liaison	Bill Barnes	william_barnes@hcpss.org
Government Relations Rep.	Nicole Paris	Nicole.Paris@fcps.org
Member-at-Large	Crystal McCubbin	cmccubbin@bcps.org
MSDE Representative	Marci Goldman-Frye	mfrye@msde.state.md.us
MCTM Conference Chair(s)	Glenn Hyman Leanne Voos	ghyman@bcps.org lvoos@bcps.org

The Maryland Council of Teachers of Mathematics is an affiliate of the National Council of Teachers of Mathematics. Membership in the MCTM is open to all persons with an interest in mathematics education in the state of Maryland. To become an MCTM member, please visit our website: https://www.marylandmath.org/membership/join .

Furthermore, the MCTM Board invites all members to become actively involved in our organization. To become involved, please contact one of the officers listed above. We would love to hear from you!

Maryland Council of Teachers of Mathematics
c/o Ed Nolan
10299 Huron Trail
New Market, MD 21774

MCTM Mission Statement: The MCTM is a public voice of mathematics education, inspiring vision, providing leadership, offering professional development, and supporting equitable mathematics learning of the highest quality for all students.