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Articles

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REVIEWERS

The individuals below have given their time and expertise to read and review manuscripts submitted for this edition of the *Banneker Banner*. We are very grateful for their help.

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Banneker Banner Submission Guidelines

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. To submit an article, please attach a Microsoft Word document to an email addressed to strickland@hood.edu with "Banneker Banner Article Submission" in the subject line. Manuscripts should be original and may not be previously published or under review with other publications. However, published manuscripts may be submitted with written permission from the previous publisher. Manuscripts should be double-spaced, 12 point Times New Roman font, and a maximum of 8 pages. APA format should be used throughout the manuscript with references listed at the end. Figures, tables, and graphs should be embedded in the manuscript. As the Banner uses a blind review process, no author identification should appear on manuscripts. Please include a cover letter containing author(s) name(s) and contact information as well as a statement regarding the originality of the work and that the manuscript is not currently under review elsewhere (unless accompanied by permission from previous publisher). If electronic submission is not possible, please contact the editor to make other arrangements. You will receive confirmation of receipt of your article within a few days, and will hear about the status of your article as soon as possible. Articles are sent out to other mathematics educators for anonymous review, and this process often takes several months. If you have questions about the status of your article during this time, please feel free to contact the editor. Please note that photographs of students require signed releases to be published; if your article is accepted, a copy of the release will be sent to you and it will be your responsibility to get the appropriate signatures. If you would like a copy of this form at an earlier time, please contact the editor.

Message from the President

Andrew Bleichfeld

Summer sure came and went fast. That's what happens when the last day of school is the third week of June, and then I worked summer inservice days until July 1^{st}, and then had another on July 15^{th}, and then started soccer meetings on August 11^{th} with practices starting on August 12^{th}. Geesh.

This time of year is the busiest for me. My Department Chair position means that I am responsible for helping everyone in my department get started with the right curricula, books, supplies, etc. <<Speaking of books, I ordered 35 student books and two teacher's editions for our new Community College class we are offering at our high school. The publisher sent 35 teacher's editions and 2 student books. Anyone notice something wrong with that order?? I guess no human checks the logic of that order before it is shipped. And, by the way, it was not my fault that the order was backwards!>>

I also coach the varsity boys soccer team here so practices and games take up every afternoon and many evenings.

My position as President of MCTM requires me to ensure that our many professional development opportunities, including our conference, are run effectively. This means a lot of emails and meetings.

And, oh yes, I also have to prepare lessons for my own classes that I teach!

Many people often ask me how I can juggle it all. How can I handle so much? I enjoy teaching. I enjoy coaching. I enjoy department chair leadership. And I certainly enjoyed serving as your President.

Please know that this organization is in good hands with our incoming President, Jen Novak, taking the reins. And as I have said before, the wonderful people on the MCTM Board of Directors made much of my job seem almost easy. ☺

Algebra: Should it be a Graduation Requirement?

By Jamie Benson, Frederick County Public Schools

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

Over the years, high school graduation requirements in the United States have become increasingly more difficult. Students today must take many more classes in the core subjects (English, Mathematics, Science, Social Studies) than students did even 10 years ago. The rigor of these courses has also increased, holding our students to higher expectations than in years past. The mathematics curriculum alone has undergone considerable changes, including the current implementation of the new Common Core State Standards for Mathematics (CCSSM, 2010). One of the most significant of these changes is the restructuring of the Algebra I curriculum to include more upper-level concepts, together with the push for students to take it at an earlier age. Competition around the globe is no doubt a contributor to this gradual increase in coursework and difficulty level, as the United States continues to significantly underperform many countries in mathematics. While it is important for our students to strive for excellence and hold themselves to high standards, is it necessary to subject all students to an increasingly more complex mathematics curriculum in order to earn a high school diploma? Should every student be required to pass Algebra I in order to graduate?

Algebra I was not always a graduation requirement throughout most of the United States. It was not until the 1990s when most states started requiring students to pass Algebra I in order to graduate. In fact, only 60 years ago, not even all students were required to take a mathematics course in high school. Layton (1954) studied the mathematics graduation requirements for the 48 states[1] (not yet including Alaska and Hawaii) and the District of Columbia from data provided by the state departments of education in 1952. Of the states that implemented a four-year high

[1] It is important to note that one state was not included due to insufficient data and Layton (1954) refers to the District of Columbia as a state in order to employ more simplified wording.

school program, he discovered that only 57% required some mathematics instruction for graduation and only three states required an algebra course.

According to the 2014 list of state graduation requirements from Achieve, Inc, 37 states[2] and the District of Columbia now require students to take at least one course that focuses on algebraic skills in order to earn a diploma. Usiskin (1980) explained that a typical 1950s college-bound student may have taken a mathematics course load that included two years of algebra, one and a half years of geometry and a half year of trigonometry. But when the Commission on Mathematics of the College Entrance Examination Board (CEEB) released a report in 1959 with their proposal for what college-bound students should be studying in high school, the new curricula became overcrowded. Some of the recommendations, among others, were that inequalities and deductive reasoning be added to Algebra I and trigonometry and complex numbers be added to Algebra II (Usiskin, 1980). There were several other additions to each of the existing curricula, but no deletions. “Nowhere did the CEEB commission report recommend that more hours be devoted to mathematics instruction each week” (Usiskin, 1980, p. 413). Students were being expected to learn more material in the same amount of instructional time.

The 37 states in 2014 that required an algebra course for graduation required that students pass the minimum of Algebra I, though 19 of those states also required students to pass Algebra II. According to an article by the Washington Post, “Of all of the classes offered in high school, Algebra II is the leading predictor of college and work success, according to research that has launched a growing national movement to require it of graduates" (Whorisky, 2011). More

[2] Some states' graduation requirements were not specified in this list, so the number of states that require algebra could be more than 37.

states used to require Algebra II, but have since removed that requirement due to a decrease in student achievement. "Some worry that Algebra II requirements are leading some young people to quit school" (Whorisky, 2011). Students do not only struggle with Algebra II; struggles with passing Algebra I are also leading students to dropout. In the fall of 2004, in the Los Angeles School District, "48,000 ninth-graders took beginning algebra; 44% flunked, nearly twice the failure rate as in English. Seventeen percent finished with D's. In all, the district that semester handed out Ds and Fs to 29,000 beginning algebra students…Among those who repeated the class in the spring, nearly three-quarters flunked again." (Helfand, 2006). Repeated failure is harmful to students' self-esteem and is certainly a primary reason why they drop out of high school. Former Los Angeles schools superintendent Roy Romer claimed, "[Algebra] triggers more dropouts than any single subject" (Helfand, 2006).

It is important to note that the mathematics course load outlined by Usiskin (1980) was intended specifically for college-bound students, but what about those students who do not wish to go to college? Chazan (2008) recalled, "In the not-too-distant past, say twenty years ago, algebra was seen as abstract mathematics suitable only for students who were developmentally ready and college intending" (p. 20). It was soon believed that algebra was a subject that all students should learn, whether they intended on going to college or not, but the curriculum was not adjusted to reflect this change. Students who had no intention of pursuing postsecondary education were now expected to master the same material as those students who were intending on going to college. This increased expectation puts unnecessary pressure on students who would prefer to enter the workforce after high school by requiring them to learn algebraic skills that they will likely never see again. Hacker (2012) stated, “To our nation’s shame, one in four ninth graders fail to finish high school…Most of the educators I’ve talked with cite algebra as the major academic reason” (para. 5). Students who do not possess the internal motivation for learning algebra – the same students who, most likely, have no interest in continuing their education after high school – may not see the need to finish high school at all, viewing algebra as too much of a struggle and an unnecessary course for their future success.

If Algebra I is to remain a graduation requirement, it is important to discuss the best time for students to learn it. Originally, before Algebra I was a graduation requirement, the only students who enrolled in the course were those on the college track. The course was typically taken during tenth grade or later, but was then pushed back to ninth grade to allow college-bound students to take more upper-level mathematics courses before graduating. Once Algebra I became a graduation requirement, ninth grade remained the time when students were expected to take the course, even those students who were not preparing for college. Now, some believe that Algebra I should be an eighth grade course and insist on pushing it back to the middle school. According to Vigdor (2012), “In the mid-1980s, about one student in six took Algebra I in middle school. In more recent years, the national average has been closer to one-third, doubling over the course of a generation” (p. 10). It is believed that exposing students to algebra at a younger age will provide a stronger mathematics background and produce more high-achieving students. However, this assumption has been disproven and, in fact, may actually lead to lower test scores and a decrease in mathematics achievement over time.

Global competition is one reason behind the push for Algebra I to be taught in middle school. Vigdor (2012) stated, "American students test poorly in mathematics compared to those in other developed—and in some cases, less developed—countries" (p. 1). In many of those countries, algebraic concepts are studied earlier than at the high school level, so, naturally, the United States feels pressured to compete with these nations. In 2012, the Programme for International Student Assessment (PISA) survey assessed the ability levels of 15-year-olds worldwide in reading, mathematics and science (focusing particularly on mathematics). Of the 34 countries that participated, the U.S. performed below average, ranking in at 27[3]. Researchers recognize that the Unites States must reflect on (and possibly restructure) its education system if it wants to remain competitive with other top nations whose students consistently and significantly outscore U.S. students. One idea to address the issue is to expose students to Algebra I at an earlier age to give them more opportunities to take advanced mathematics courses in high school. However, research from Charlotte-Mecklenburg, North Carolina – one of the nation's largest and most successful school districts – showed that students who took Algebra I at an accelerated rate scored significantly worse (thirteen percentile points lower) on the state's end-of-course test compared with students who took Algebra I on a regular schedule (Vigdor, 2012). That is, students who took Algebra I on the normal track – in high school – performed better.

Other concerns arise from the argument of making Algebra I an eighth grade course. Chazan (2008) explained, "As more students take algebra before ninth grade, more middle school teachers with elementary school (K-8) certification are being asked to teach algebra, a domain of mathematics they were not prepared to teach" (p. 23). If the intent is to teach algebra to middle school students, then it is imperative that middle schools employ highly qualified mathematics teachers. It is detrimental for students to be placed in Algebra I classes instructed by teachers who are not well versed in the subject matter. This is especially true for younger students, whose minds are still so pliable and impressionable that we do not want them to develop a negative view of mathematics at such an important age.

As Vigdor (2012) explained, "Can we really think of an algebra course offered to every eighth grade student as the intellectual equivalent of a course that was offered only to the top quarter of students, typically in tenth grade or later, sixty years ago?" (p. 11). The Common Core State Standards for Mathematics (CCSSM, 2010) have set out to enhance the rigor of the Algebra I curriculum, resulting in increased stress among high school students seeking a diploma, not to mention the impact it could have on eighth graders. In order for the United States to see an increase in its number of high-achieving, college bound, STEM career searching students, and to raise its global ranking in mathematics performance, we must develop a better plan than simply pushing Algebra I back into the middle schools.

According to Hacker (2012), during a typical American school day, about six million high school students and two million college freshmen struggle with learning algebra. Too many of the students in these classrooms are expected to fail, so Hacker asked "Why do we subject American students to this ordeal? I've found myself moving toward the strong view that we shouldn't" (para. 1). Algebra is often referred to as the "gatekeeper" to higher learning and a strong predictor of success in both college and the workforce. But Noddings (2012) explained, "The U.S. Bureau of Labor Statistics informs us that most of the job openings in the next decade

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[3] Rank 27 is the best estimate, but due to sampling and measurement error the rank could be between 23 and 29. This is in part because students from the most disadvantaged schools were over-represented in the U.S. sample, whose performance is relatively low.

will be in occupations that do *not* require a college education; reasonably, then, we should consider how best to provide for students whose interests are not primarily academic" (para. 9). If most of the country's future jobs will not require college educated employees, must we require all students to pass a course that was once reserved only for the college intending?

Graduation requirements have been quickly changing over the last 60 years in an effort to ensure college and career readiness for all students. As the future of the United States continues to change, it is essential that its education system continues to change as well. We must work toward developing graduation requirements that meet the needs of individual students, rather than imposing a standard set of requirements for all. Students should be eager and excited to study mathematics and feel that they are learning skills that will be of great value to them after high school. These necessary skills will vary from student to student, where some will require Algebra I and some will not. There is no need to continually raise the bar for earning a high school diploma, especially when so many students believe the bar to be unattainable and end up dropping out of school. In a nation comprised of such a variety of occupations – both for the college and non college educated – and the obvious gains that differentiation can bring, the academic goal should be excellence for all students, not algebra for all students.

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Project-Based Learning in a Middle School Teacher Education Problem Solving Course

By Diana Cheng, Shannon Moore, Jennifer Wong, Towson University

The work of a teacher involves analyzing student reasons to problems: deciding whether the students are correct or incorrect in their approaches, interpreting this work to see where students might have gone astray or have limited conceptions, and categorizing these limited conceptions to be able to address them with students. In a problem solving mathematics content course for middle school pre-service teachers, the first author of this paper provided undergraduate students with a project-based learning (PBL) opportunity so that they could experience doing the genuine work of a teacher. In this article, we describe this PBL assignment, provide an example of one team's project completed by the second and third authors of this paper, and discuss this sample project in light of the Common Core State Standards.

Project- Based Learning (PBL) for pre-service teachers

According to Blumenfeld et. al. (1991), PBL involves a driving task that allows students to explore problems and draw on interdisciplinary concepts. This task should be open-ended and allow for multiple approaches. In pursuing this task, students should be required to create artifacts, which are demonstrations of their work and are continuously subject to revision. Projects have the potential to increase student motivation because students are solving authentic problems, collaborating with their peers, and developing real solutions to these authentic problems.

The goal of the project in this course was to mimic the work of a teacher – a teacher not only needs to be able to solve interesting math problems but also design such problems and try to understand how others approach the problems to inform instructional choices. The open-ended driving task for this project was the following: “Design an original, middle school level mathematics modeling problem to solve. This problem should allow for multiple solution paths and should not specify a specific pathway to a solution. Ask your classmates to solve the problem in at least two different ways. Examine your classmates' solutions to your problem. Classify all of the solutions by type of solution method used. For the incorrect solutions, determine what the error(s) were and classify the solutions based on error type. Write a report clearly explaining the different types of correct solutions and incorrect solutions.”

Sample PBL task created by middle school pre-service teachers

In response to the above prompt, the second and third authors of this paper created the following original problem: “Each of the 26 letters of the English alphabet is represented by a number, according to Table 1.

Number	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Letter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Table 1. Key of alphabet letter and number representations.

A word is then represented by multiplying the numbers which represent each letter of the alphabet. For example, DOG is represented by $(4 \times 15 \times 7) = 420$. Find a word that can be represented as 7560 and explain how you found it. All numbers found as factors could be reordered to create a word. Is it possible to find multiple words? Explain why or why not.”

During the semester, the students in this course were introduced to cryptography, the encrypting and decrypting of messages, via multiple experiences. Students toured the National Cryptologic Museum in Fort George G. Meade, Maryland adjacent to the headquarters of the National Security Agency (NSA, 2012). There, they completed a mathematical scavenger hunt which introduced them to historical applications of cryptology: people who contributed to cryptology and national defense, the tools that they developed, and the locations in which these tools were used. For homework, students read a Mathematics Teaching in the Middle School article (Chua, 2008) focusing on the use of linear encoding processes and the use of inverse functions to decrypt messages, and completed the article's accompanying student worksheets. Students also completed an in-class activity on encrypting and decrypting messages using matrix addition and the Vigenère cipher. Students were familiar with the idea that modular arithmetic can be used to represent letters (e.g., since 27 mod 26 = 1, and since the number 1 represents A, the number 27 would represent the letter A).

The problem does not describe a valid method of encryption, since encryption methods must be bijections. In the problem, each output such as 7560 does not have a unique input, or in other words, there are multiple words that would yield a product of 7560. However, the problem is an interesting interdisciplinary application of the mathematical concept of factors and the English language idea of anagrams. The problem is cognitively challenging (Smith & Stein, 1998) because it requires students to apply their knowledge of factoring and cryptography to a task requiring non-algorithmic thinking.

Peer solution analysis for task

Five peers of the middle school pre-service teachers who designed the problem were surveyed. All of the students who completed the activity understood that factoring was needed and they proceeded to factor 7560 in various ways. Most used a factor tree when factoring. Some of the students' productive strategies include the following:

- When factoring, students deliberately attempted to yield results with numbers representing vowels since most words in the English language include vowels.

- Since 1 is the multiplicative identity, the letter "A" (which corresponds to 1) can always be used an unlimited number of times to the existing letters found. For instance, once you have found that $7560 = 42 \times 12 \times 3 \times 5$ yielding the letters P, L, C, and E, you can form the words "PLACE" and "PALACE" by adding in one and two A's respectively.
- Because of the commutative property of multiplication, letters can be arranged in different combinations to form distinct words. For example, $7560 = 12 \times 9 \times 1 \times 70$ (LIAR) and $7560 = 12 \times 1 \times 9 \times 70$ (LAIR), form two words using the same letters.

Common issues that we observed in student work which led to incorrect solutions or a lack of solutions include the following:

- Some students did not consistently factor accurately. This resulted in students finding words that they believed could be formed but in fact could not, as well as students' inability to create words that could otherwise have been formed if factoring was completed properly.
- Some students made mistakes when performing modular arithmetic, especially if they did not realize that the modulus to be used was 26.
- Some students aimed to find words with too large of a number of letters. The more factors other than 1 that are found, the closer they are to the beginning of the alphabet, and it is difficult to form multiple words with just letters from the beginning of the alphabet. An extreme case is finding the prime factorization of 7560, $2^3 \times 3^3 \times 5 \times 7$. The letters corresponding to these prime factors are B, B, B, C, C, C, E, and G. If we only use the letters corresponding to these prime factors, there are too many consonants and only one vowel, making it impossible to form a word.

Some sample words that can be formed are listed in Table 2. These include words found by the surveyed students as well as those found by the authors of this paper.

Multiplication expression equivalent to 7560	Corresponding word (definition also provided, if not a common English word)
1 × 7560, 280 × 27	AT
1 × 840 × 9	AHI (yellowfish tuna)
1 × 14 × 540, 1 × 378 × 20	ANT
2 × 27 × 4 × 7 × 5	BADGE
2 × 9 × 420	BID
2 × 9 × 12 × 7 × 5	BILGE (the outer area of a ship's hull)
4 × 1 × 210 × 1 × 9	DABAI (a fruit indigenous to Borneo)
4 × 135 × 14	DEN
5 × 756 × 2	EBB
135 × 14 × 4	END
6 × 12 × 15 × 7	FLOG
6 × 21 × 5 × 12	FUEL
7 × 1 × 2 × 1 × 540	GABAT (a city in France)
7 × 9 × 6 × 20	GIFT
840 × 9	HI
140 × 1 × 54	JAB
12 × 1 × 9 × 70	LAIR
168 × 5 × 9	LEI
12 × 9 × 1 × 70	LIAR
168 × 9 × 5	LIE
12 × 9 × 14 × 5	LINE
42 × 1 × 4 × 45	PADS
42 × 1 × 12 × 1 × 3 × 5	PALACE
42 × 12 × 1 × 3 × 5	PLACE
18 × 1 × 420	RAD
18 × 21 × 4 × 5	RUDE
18 × 21 × 20	RUT
540 × 1 × 7 × 1 × 2	TAGAB (a city in Afghanistan)
540 × 1 × 14 , 20 × 1 × 378	TAN

Table 2. Sample solutions to problem.

Additionally, the authors of the paper found it helpful to list out all of the factors of 7560 and their corresponding letters (see Table 3). This table was not provided to the surveyed students.

Factor (F) of 7560	F mod 26	Corresponding letter
1	1	A
2	2	B
3	3	C
4	4	D
5	5	E
6	6	F
7	7	G
8	8	H
9	9	I
10	10	J
12	12	L
14	14	N
15	15	O
18	18	R
20	20	T
21	21	U
24	24	X
27	1	A
28	2	B
30	4	D
35	9	I
36	10	J
40	14	N
42	16	P
45	19	S
54	2	B
56	4	D
60	8	H
63	11	K
70	18	R
72	20	T
84	6	F
90	12	L
105	1	A
108	4	D
120	16	P
126	22	V
135	5	E
140	10	J
168	12	L
180	24	X
189	7	G
210	2	B
216	8	H
252	18	R
270	10	J
280	20	T
315	3	C
360	22	V
378	14	N
420	4	D
504	10	J
540	20	T
630	6	F
756	2	B
840	8	H
945	9	I
1080	14	N
1260	12	L
1512	4	D
1890	18	R
2520	24	X
3780	10	J
7560	20	T

Table 3. Factors of 7560 and their corresponding letters.

Common Core State Standards addressed by PBL activity

A number of Common Core State Standards for Mathematics (CCSSI, 2010a) and English Language Arts (CCSSI, 2010b) can be addressed through this activity developed by middle school pre-service teachers. The most salient Common Core State Standards for Mathematics (CCSSI, 2010a) which are addressed in this activity are listed in Table 4. The Common Core State Standards for English Language Arts (CCSSI, 2010b) recommend that students in grades 4 through 8 learn to determine the meanings of words through consulting reference materials such as dictionaries. While students are solving anagrams derived from the factors of 7560, they could be allowed to use dictionaries to determine whether the letters they find form words.

Common Core State Standards: Standards for Mathematical Practice	
MP1: Make sense of problems and persevere in solving them.	Students need to understand the instructions of the problem, understand what a solution looks like, and find a way to determine solutions. In order for a word to be a proper solution, its letters need to be able to be represented as numbers whose product is 7560. In order to obtain numbers whose product is 7560, students must first factor 7560 and find the corresponding letters to these factors. Then they can make judgments through referencing a dictionary as to whether the letters found form words. It is not easy to go through this process of finding solutions, so students must persevere if they would like to find multiple solutions.
MP3: Construct viable arguments & critique the reasoning of others.	Middle school pre-service teachers evaluated the solutions which their peers developed, as well as analyzed their peers' work in terms of whether solution methods would work if applied correctly. In the middle school classroom, a teacher could have students similarly determine whether or not their classmates' methods and solutions are correct.
MP5: Use appropriate tools strategically.	Calculators could be used to help students find factors, but students need to know how a calculator can help (e.g., what would a quotient look like in a calculator if you divide 7560 by a number which is not a factor?). Dictionaries could also be useful in this activity, but students need to form combinations of letters which they think are words before they can look them up in the dictionary to verify whether or not these are in fact words.
MP7: Look for and make use of structure	Students can observe that using the commutative property of multiplication and the identity property of multiplication can be useful in solving this problem.
Common Core State Standards: Content Standards	
4.OA.B.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors.	Students need to find factor pairs for the whole number, 7560. Middle school students could factor a smaller number such as 60 (producing words such as BAD, DO, ADO, CAT, and AHA) or 84 (producing words such as FAN, CAB, and LAG).
6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.	Students need to divide a multi-digit number, 7560, in an efficient manner in order to determine its factors.

6.EE.A.2.a: Write expressions that record operations with numbers and with letters standing for numbers.	To perform modular arithmetic with modulus 26, some students wrote "(Factor) – 26n" whereby they attempted to find whole number n as the number of times that 26 was to be subtracted from the factor

Table 4. Common Core State Standards for Mathematics addressed in this activity.

Some potential extension questions to this problem include the following:

- What is the longest word which can be represented by 7560 that can be formed using the process described?
- How does the choice of letter-number representations affect the possibilities of words that emerge? (e.g., if we picked a different letter-number table in which E = 1, would there be more words that could be formed?)
- If you were to come up with a different number other than 7560 to use in this scenario, what would you use and why?

As we have shown in this article, embarking on a PBL activity can allow pre-service teachers to experience the genuine work of a teacher in creating a mathematical problem and analyzing student solutions to this problem. In doing so, the pre-service teachers and their peers addressed many middle school level content and practice standards. This paper, reporting the task and different solution methods, is itself is an artifact of the PBL experience. The paper is based on the report that the second and third authors developed as part of their course grade and expanded upon by the first author to illustrate how PBL could be incorporated in the instruction of pre-service teachers.

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What’s the difference? Why Asians excel at math

By Lori Wilson

Research reported in this paper was completed in a capstone project for the master’s degree in mathematics education in the Graduate School of Hood College.

I had a friend ask me in the spring, “Is it true that American students are much worse at math than students in Asian countries? What’s so different?” This inspired me on to research this topic to be able to give a solid answer to this question, specifically looking at China, Japan and Singapore.

In the 2012 Program for International Student Assessment (PISA), the United States ranked 27th out of the 34 Organisation for Economic Co-operation and Development (OECD) countries in mathematics. Looking at individual states as countries, Massachusetts would have ranked seventh in the world; however, 23 states would have fallen below 30th place out of the 34 countries. At the top of the PISA rankings were Shanghai-China, Singapore, Hong Kong- China, Chinese Taipei, Korea, Macao-China and Japan (OECD, 2014).

Not only is the United States lagging behind in international assessments; despite students’ graduating high school and being eligible for college, they are not ready for college-level math. According to the National Center for Public Policy and Higher Education, nearly 40% of all freshman college students must take remediation courses in math and/or English, which are not credit-bearing. Of students entering two-year colleges, it is up to 75% who need remediation in math and/or English (The National Center for Public Policy and Higher Education, 2010).

It has been clear that change needs to occur, which is why the Common Core State Standards (CCSS) were created. The goal of CCSS is to prepare students to take credit-bearing introductory courses at two- or four-year colleges or enter the work force. The Common Core State Standards put us on the same playing field as high-achieving Asian nations, yet even with similar curriculums, we still lag behind in cultural attitudes, teacher quality and professional development (CCSS, 2014).

Curriculum and Teaching Methods

Prior to the development of the Common Core State Standards, beginning in 2009, standards widely varied from state to state. States often had general guidelines that the almost 16,000 U.S. school districts used to design their curriculum standards. In comparing American education to top performing countries on international assessments, researchers noted that the top performing countries had national curriculums. They also came to the conclusion that the United States curriculum needed to become more focused and coherent. The previous curriculums in America were often described as "a mile wide and an inch deep." They were not focused, having more topics at each grade level than any other country. In addition, the state standards were often highly repetitive, not demanding, and incoherent. There was no logical structure to the curriculum as it jumped from basic to advanced topics (CCSS, 2014; NCES, 1997; Schmidt, Houang and Cogan, 2002).

International benchmarking did contribute a significant role in the creation of the Common Core Standards. The creators of Common Core looked to the math standards of top performing countries on international assessments. Schmidt and Huoang compared the focus, content and rigor of the Common Core Standards to the top performing countries on the 1997 Trends in International Mathematics and Science Study (TIMSS) and concluded that there is almost an 85% consistency between Common Core Standards and the standards of the A+ countries (CCSS, 2014; Schmidt & Huoang, 2012).

Achieve (2010) compared CCSS to both Singapore's math standards and Japan's Standard Curriculum. They found similar rigor and that any content discrepancies in curriculum for each grade band typically fell within a year of each other.

The United States is also seeking to emulate a focus on conceptual understanding. China, Japan and Singapore all encourage multiple approaches to any one problem. The focus is not on the right answer, but the mathematical thinking and conceptual understanding behind it. Encouraging multiple solutions allows students to see the advantages and disadvantages of different approaches. The Common

Core Standards are shifting from teaching students procedurally, focusing on getting the right answer, to teaching conceptually, where students need to apply a mathematical idea, not procedure, to a problem (Xu, 2010).

Cultural Beliefs

In China, Japan and Singapore alike, education is highly valued and seen as the path to success in life. As a result, parents are dedicated to the education of their children. Once students leave school at the end of the day, the parents take responsibility to oversee their education, from helping them with homework to hiring tutors or enrolling them in other academic services. Because the families put this energy into helping their students, the students want to do well to make their families proud. All three countries have important exams the students needed to do well on to get accepted not only into university, but also into upper secondary school. These exams serve to focus the students. In America parents are not involved in the same way. As the Michigan Department of Education says, “Lack of parent involvement is the biggest factor facing public schools” (2002, p.1) (NCEE, 2014).

Citizens of China and Japan both see success as a result of effort, not ability. Natural intelligence does not limit achievement because diligence and hard work can compensate for lower natural ability. In America, ability is seen as a bigger contributor to success than effort. Because ability is often out of one’s control, seeing success as a result of ability diminishes students’ motivation if they feel like they have low ability. They are more likely to give up. In America, we tend to focus on each student’s individuality and will place a student in an honors track to maximize his or her individual potential. This focus on individuality and ability extenuates the performance gap. On the other hand, focusing on effort increases hope and persistence in learning math because success is within one’s control (Newton, 2007; Xu, 2010).

Teacher Quality

According to several measures, teachers are of a higher caliber in China, Japan and Singapore than in America. In Japan and Singapore specifically, getting admitted into teaching programs is highly competitive, resulting in teachers’ being selected from the top students. In Japan only 13% of applicants are accepted into teaching education programs and in Singapore, only one in every eight applicants is accepted. On average, teaching in America does not draw high-achieving students. Fortmann, Eisenkraft & Sevian found that “Teachers are increasingly drawn from the bottom quartile of college students as new career options have opened up over recent decades for minorities and women” (2005, pp. 2-3). College graduates majoring in education have lower ACT/SAT scores than those majoring in the arts and sciences (NCEE, 2014, Whitehurst, 2003).

Teachers in these three Asian nations also have strong math backgrounds comparable to having a major in math. In Japan, 98% of teachers hold a certificate in the content area where they teach. In Singapore, 80% of applicants to the teaching program already have a bachelor’s degree in the area they want to teach. In China as well, a greater percent of math and science teachers have degrees in their content area than in America. American math teachers do not all have a strong math content background. A 2007 report by the National Academies reported that 69% of fifth- through eighth-graders in America are taught math by teachers without a degree or certificate in math. Even in high school, 31% of students have math teachers without a degree or certificate in math (NCEE, 2014; The National Academies, 2007; Schmidt et al., 2002).

Teachers are highly respected in Asian nations, however in America, teaching is not considered an esteemed job as seen in the salary. According to a study done by the National Association of Colleges and Employers, the average starting salary in America for 2013 graduates was $40,590 in education, $55,140 in business, $59,084 in computer science and $62,654 in engineering (2014). Singapore, on the other hand, strives to make their teachers’ salaries competitive with other professions in order to get and keep quality teachers. In Japan, beginning teachers are paid the same amount as beginning engineers. In China, teachers’ pay varies between urban and rural areas. In large cities, teaching salaries are very competitive. Teaching in America does not have a competitive salary to attract the most talented students to become teachers (NCEE, 2014).

Teaching Profession

Teachers in these top-performing countries not only have better content knowledge entering the profession, but they also have better professional development once they are teaching. Because top performing countries have national curriculums, professional development for teachers is focused on the specific grade level content they are teaching and how to teach it. Rather than being specific and content based, most American professional development tends to be in the form of generic workshops. The Asian nations do a much better job preparing their teachers for their curriculum. Teachers in Singapore have solid professional development on the curriculum they are teaching. The Ministry of Education is extremely careful in implementing new policies to assure all the necessary changes are in place and proper training is given. Similarly, the Chinese Ministry of Education has a rule that teachers must receive professional development and training prior to implementing a new curriculum. A July 2014 *New York Times* article (Green, 2014) describes the fact that, with Common Core, as with previous reform attempts, there is no good system in place to train teachers to teach the new standards. We might have international benchmarked standards, but without successful implementation, it will simply backfire (NCEE, 2014; Paine & Fang, 2006).

Along with professional development, these Asian nations have tools to help implement the curriculum. China, Japan and Singapore all have textbooks approved by the Ministry of Education or created by the Ministry of Education. No such list exists in the United States. Now all kinds of education materials are marketed as "Common Core Aligned"; however, a study by William Schmidt, a professor of statistics and education at Michigan State University, showed that some of these books were almost identical to their prior textbooks. Morgan Polikoff, a professor at the University of Southern California, found that in the Common Core textbooks used in Florida, 15-20% of the material was extra (not in Common Core), 10-15% of Common Core material was not in the books, and 60-70% of each book was identical to the pre-Common Core versions. Polikoff also stated what was missing of these textbooks was the higher level thinking questions, one of the main goals of Common Core (Herold & Molnar, 2014; NCEE, 2014).

Japanese and Chinese teachers also have much more time available in their day for collaboration and preparation. According to Organisation for Economic Co-operation and Development (OECD), lower secondary teachers in the United States are teaching 54% of their total work time whereas the lower secondary teachers in Japan teach 32% of their work time. The OECD average is 42%. There is an even greater difference at the upper secondary level where American teachers spend 53% of their time teaching and Japanese teachers spend 26% (OECD average 39%). Chinese teachers only spend 40% of their time teaching and spend the remainder planning lessons, grading, studying resources and collaborating with colleagues. (Newton, 2007; OECD, 2011).

Content-wise the Common Core State Standards near the rigor of the Asian curriculum; however, the United States still lags behind in cultural attitudes, teacher quality and professional development. Is the change in curriculum enough to improve our math performance internationally? An analysis by OECD comparing CCSS and PISA "suggests that successful implementation of the Common Core State Standards would yield significant performance gains also in PISA" (2012, Country Note United States, How does PISA relate to CCSS in mathematics section, para. 2). Will those gains be enough to make us competitive with China, Japan and Singapore? If not, are we willing to devote the time and money to increase teacher quality? Do we sacrifice our focus on individuality in an attempt to become more internationally competitive in math? To what extent are we willing and able to change our education system and culture to meet these demands?

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Connecting Research to Practice: Teaching Fraction Word Problem Solving with Schema-Based Instruction

By Tricia K. Strickland and Christy D. Graybeal
Hood College

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Objective: Students will solve change problems involving fractions with like denominators with support of Schema-Based Instruction, which includes diagramming and using the FOPS strategy.

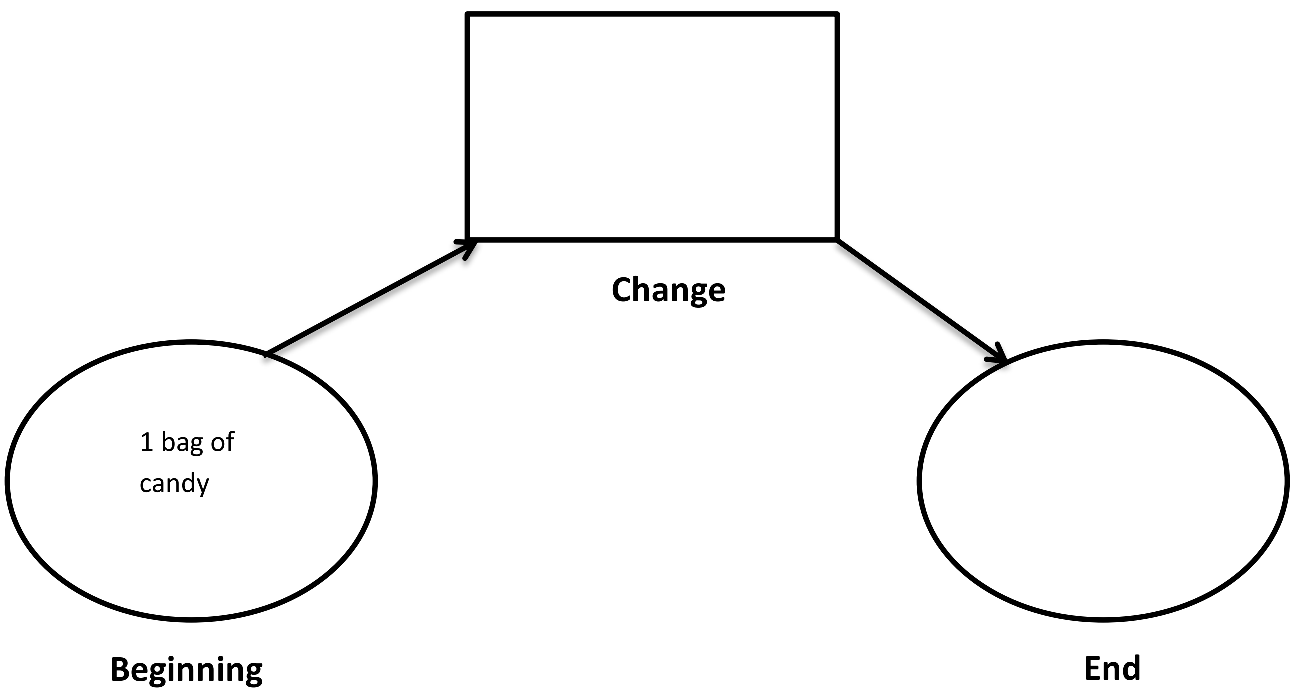
Note to teachers: Schema-based instruction (SBI) is a research-based strategy that emphasizes the mathematical structure of word problems. SBI teaches students to identify the problem structure (i.e., *change*), represent the problem in a diagram, and solve the problem. Teaching students to solve word problems based on problem structure, or situation, is emphasized in the Common Core State Standards (see p. 88 of the CCSS document at http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf). There is a plethora of research that demonstrates SBI as an effective strategy to teach students with disabilities.

SBI teaches students the structure of a word problem through a story situation. The story situation in this lesson is *Kira received one bag of candy for Halloween. She ate 1/5 of her bag of candy. Kira now has 4/5 of her bag of candy.* There is no calculation needed, the emphasis is on identifying the problem structure (*change*) and representing this situation in a diagram. Then, the teacher shares word problems from the story situation. Students are encouraged to refer back to the story situation, as the objective is to learn about the type of word problem, not to focus on calculation. Future lessons would include only word problems. Additionally, students may be initially provided the diagrams, however, they are gradually withdrawn. Students may choose to make their own diagrams or work with abstract symbols only. After students learn a variety of problem situations, they should be given an assessment which includes word problems with different structures.

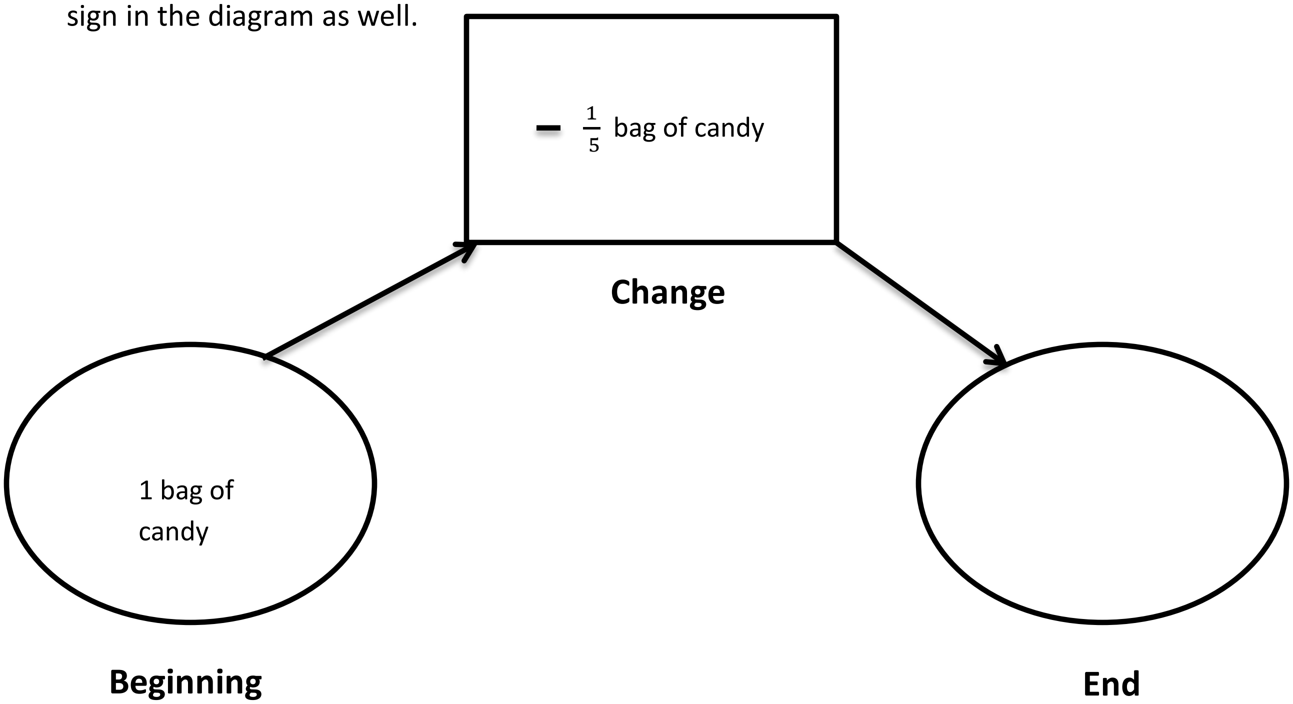
Set-up

Teacher: Today we are going to learn about change word problems with fractions. From our work with whole numbers, what are the characteristics of a *change* problem? (*Prompt as needed to have students remember that change problems have a beginning amount, a change action that increases or decreases the beginning amount, and then an end amount*.) We can organize the information in a *change* problem by using diagrams. Let's look at a *change* story situation together. Remember this story situation will not require any calculations. We are learning the structure of this type of word problem. (*Teacher gives all students the Change Story Problem worksheet.)*

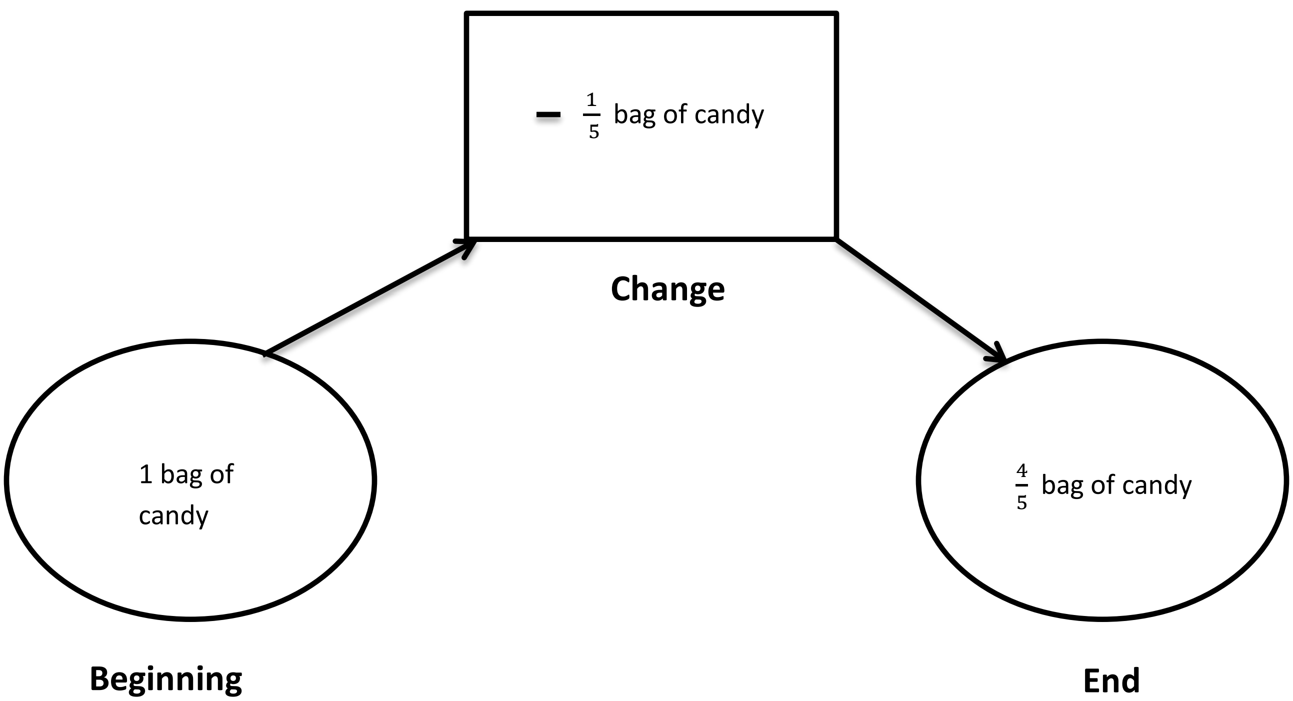
Presentation:
Teacher: Looking at our checklist (which should be displayed as well as each student having their own copy- See attached), we first need to **Find** the problem type. Follow along as I read the problem. *Kira received one bag of candy for Halloween. She ate 1/5 of her bag of candy. Kira has 4/5 of her bag of candy left.* Can I please have a volunteer to retell the story?
*Student retells the story using his or her own words***.**
Teacher: Great job retelling the story! Now how do I know if it is a *change* problem?
Prompt students to say there must be a beginning, change action, and an end.
Teacher: Now we are ready for Step 2 (point to this on the Change Problem Solving Checklist). Let's organize the information from our problem using the change diagram. To do this, we need to first reread our story to identify important information.
First, underline the label that describes the topic of the problem (bag of candy). Next circle the numbers. Ok, now that we have identified some important information, let's work on filling in our diagram. What is our beginning? (Kira has one bag of candy). Let's place that in our diagram. Remember that our whole is the bag of candy.



Teacher: What change has happened? (Kira ate 1/5 of her candy). Let's place that in our diagram. What operation will this be? Are we adding this or taking it away? How do we know what operation this is?
Since we are taking away 1/5 of the candy in the bag, we must subtract. Let's put a subtraction sign in the diagram as well.



Teacher: What is our ending? (Kira has 4/5 bag of candy left). Let's place that in our diagram.

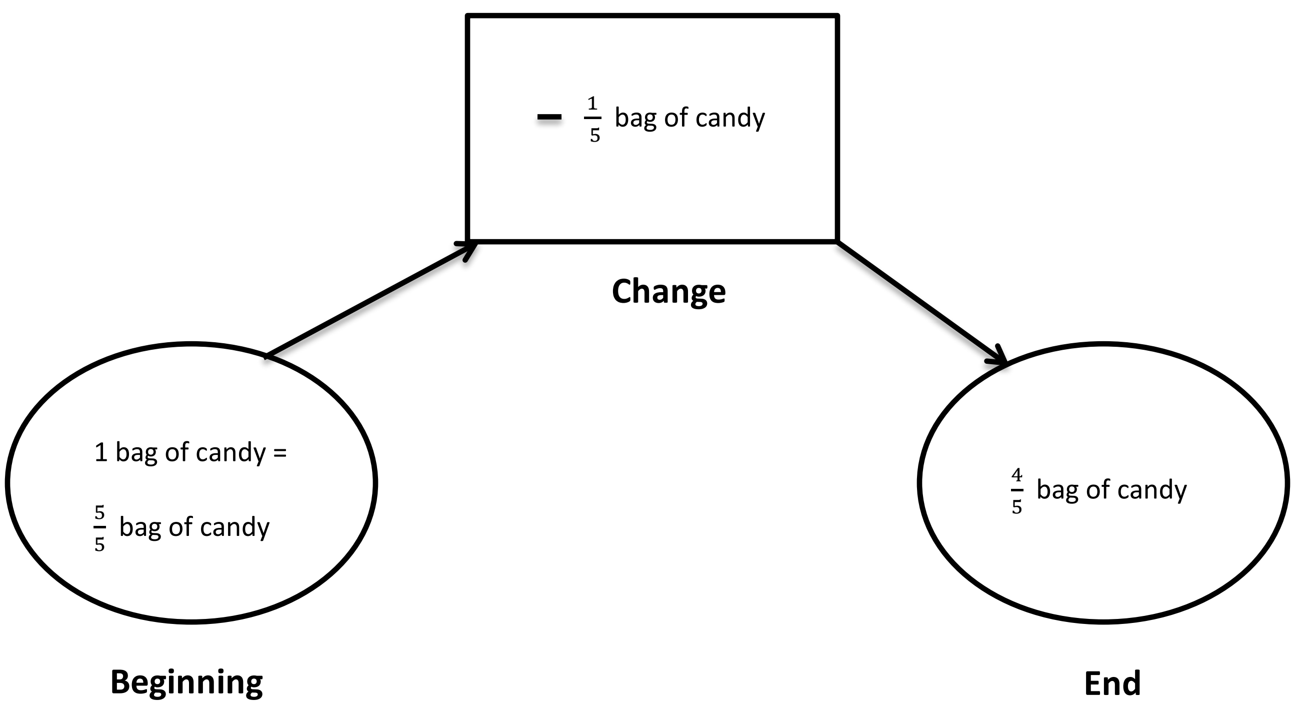


Teacher: Great work helping me fill in our diagram! Make sure your diagram on your worksheet is also filled in. Let's also make a number sentence for this. Talk with your tablemates to come up with a number sentence. ($1 - \frac{1}{5} = \frac{4}{5}$)

The teacher will now lead the students through a word problem from this story situation.

Teacher: Now let's explore some math problems that would go along with this story situation. Listen to the following problem and follow along on your worksheet. *Kira received one bag of candy for Halloween. She ate* $\frac{1}{5}$ *of her bag of candy. How much of the bag of candy does she have left?*

Let's look at Step 3 of our checklist to **Plan** to solve this problem. We need to write a number sentence to represent this problem. What is our beginning amount? (1 bag of candy). Since our change and ending amounts are in fifths, can we create a fraction with a denominator of 5 that is equivalent to 1? (5/5) Let's add this to our diagram.



Teacher: We have 5/5 bag of candy. What change is happening? (Kira ate 1/5 of the candy). How can we write this as a math sentence? Use your diagram for help.

$\frac{5}{5} - \frac{1}{5} = ?$

So we are now ready for Step 4 **Solve** the problem. What does this equal? Use your diagram from the story situation for help (4/5). So the number sentence is

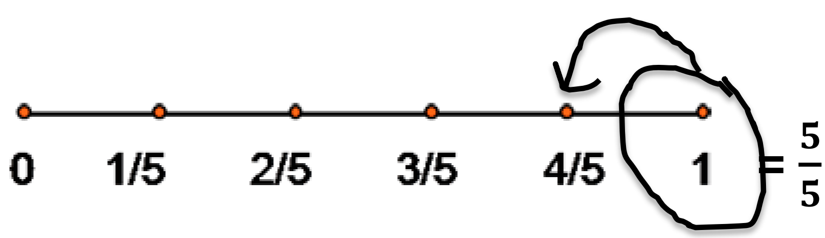
$\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$

Does this answer make sense? How do we know that $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$? Let's check our work using a number line.
(Teacher should have a number line on the board)

Teacher: Here's our number line. Where is our beginning value on this number line? (1 which equals 5/5) Let's circle 5/5.

Since we are subtracting 1/5, which way do we move on the number line? (to the left) Why? (decrease in value)

Let's move 1/5 to the left? What is our answer? (4/5)



Learning Together:
Teacher: Great job! Now you have two problems to solve using this same story (see attached). Work with your tablemates to solve these two problems on your worksheet. Remember to use your checklist as you go through the problem. Check off each step as you work. Also, although you will already know your solution because of the story, you must prove your answer is correct by using a number line, manipulatives, or another appropriate representation.

Students work in small groups to solve the problem *Kira received one bag of candy for Halloween. She has* $\frac{4}{5}$ *of her bag of candy remaining. How much of the bag of candy did she eat already?*

Teacher: Great job working with your friends! Let's share some of your strategies for answering these problems. *(Call on a few volunteers to share).*

Just for Me:

Teacher: Now you are going to complete a problem just for you. Remember to use your checklist as you go through the problem. Check off each step as you work. Also, although you will already know your solution because of the story, you must prove your answer is correct by using a number line, manipulatives, or another appropriate representation. *Give students the worksheet entitled Just for Me.*

This is also a time to provide additional support (reteaching, guided practice) to students as needed.

Assessment: Students independently complete the exit slip which the teacher collects.

Wrap it Up:

Teacher: What did we learn today? *Additional prompting questions include:*

What type of story did we work with? How can we identify a Change Story?
What did we learn about fractions? How can we rename 1 as a fraction?
How can a number line help us add and subtract fractions?

Tonight, please share with your family the characteristics of a change problem. Please make up a change problem for a family member to solve.

Notes: This lesson focuses on teaching the FOPS strategy and diagramming a story situation. Future lessons should focus on solving change word problems, without the story situation. For additional information:

Jitendra, A. (2007). *Solving math word problems: Teaching students with learning disabilities using schema-based instruction.* Austin, TX: PRO-ED.

FOPS

Change Story Checklist

Step 1: **F**ind the problem type.

- ☐ Did I read and retell the story?
- ☐ Did I ask if it is a change problem? (Did I look for the beginning, change, and ending? Do they all describe the same thing?

Step 2: **O**rganize the information in the problem using the change diagram.

- ☐ Did I underline the label that describes the context and write the label in the diagram?
- ☐ Did I underline important information, circle numbers, and write numbers in the diagram?

Step 3: **P**lan to solve the problem.

- ☐ Do I add or subtract?
- ☐ Did I write a math sentence?

Step 4: **S**olve the problem.

- ☐ Did I solve the math sentence?
- ☐ Did I write the complete answer?
- ☐ Did I check if the answer makes sense?

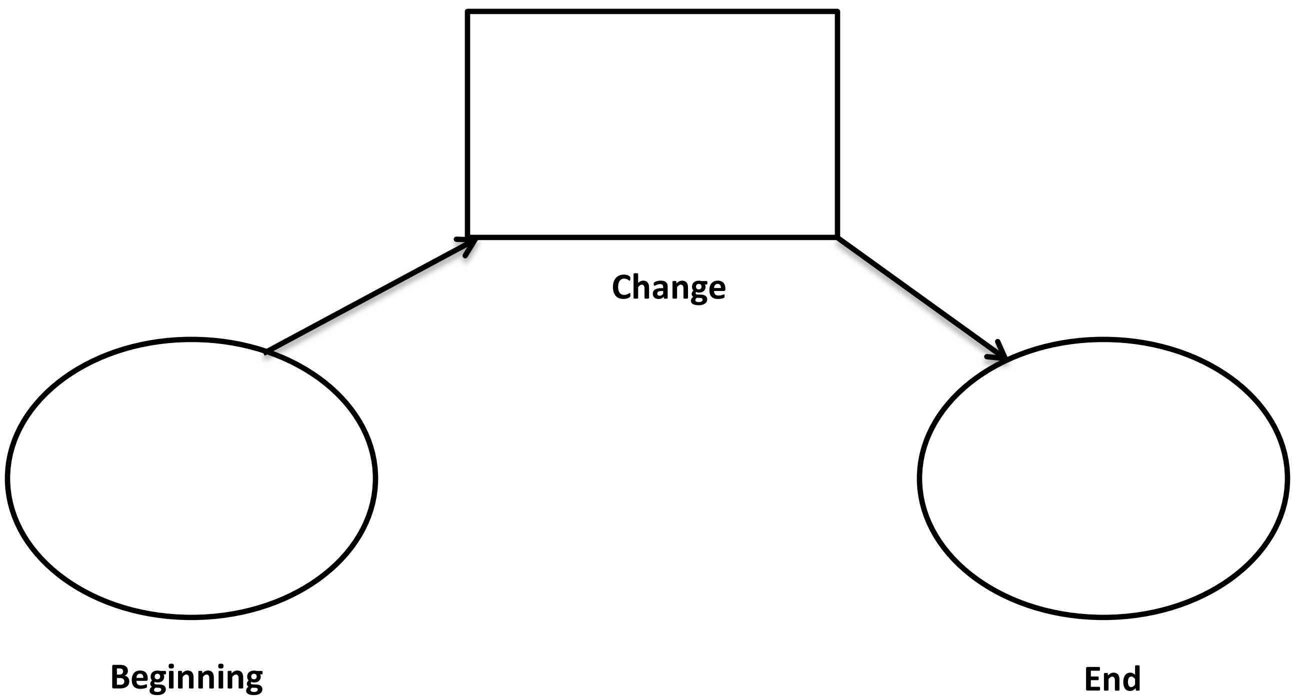
Adapted from: Jitendra, A. (2007). *Solving math word problems: Teaching students with learning disabilities using schema-based instruction.* Austin, TX: PRO-ED.

Change Story Problem

Name:____

Teacher Presentation:

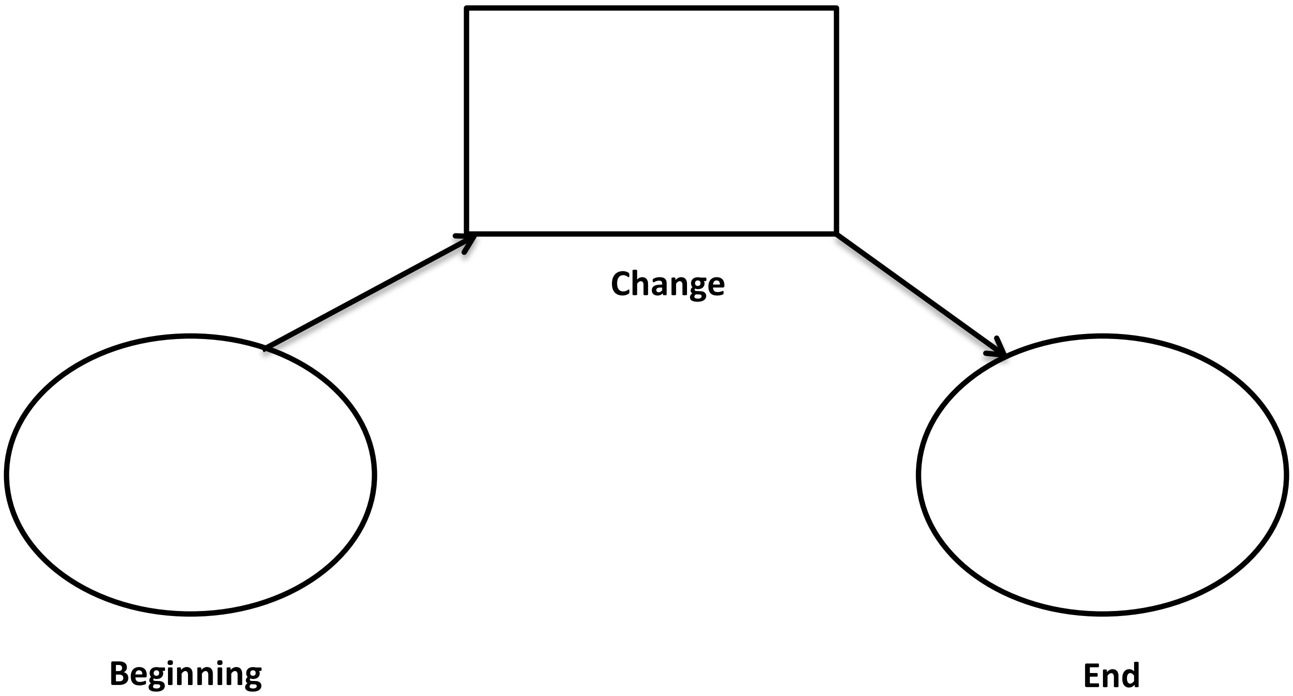
Story: Kira received one bag of candy for Halloween. She ate $\frac{1}{5}$ of her bag of candy. Kira now has $\frac{4}{5}$ of her bag of candy.



Number sentence:____

Math Problem Completed during Teacher Presentation

Kira received one bag of candy for Halloween. She ate $\frac{1}{5}$ of her bag of candy. How much of the bag of candy does she now have?



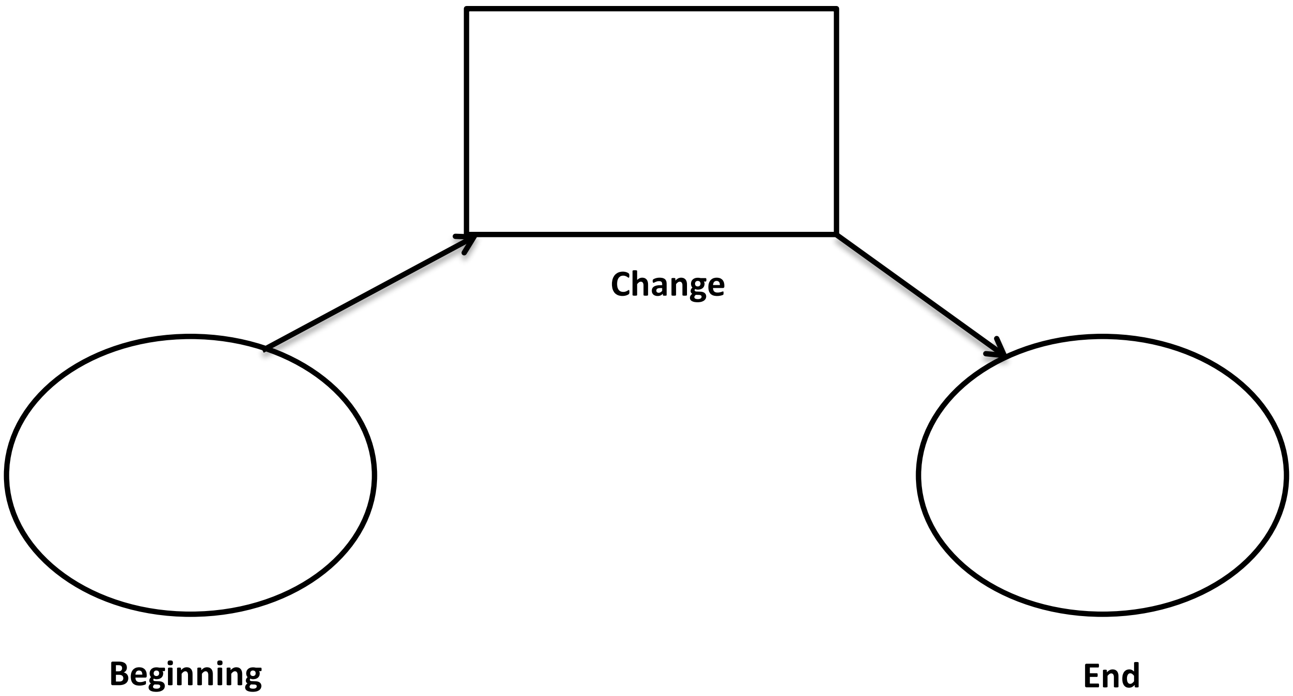
Number sentence:____

Solution:____

Prove your solution is correct in the space below.

Math Problems completed during Learning Together

Kira received one bag of candy for Halloween. She has $\frac{4}{5}$ of her bag of candy remaining. How much of the bag of candy did she eat already?

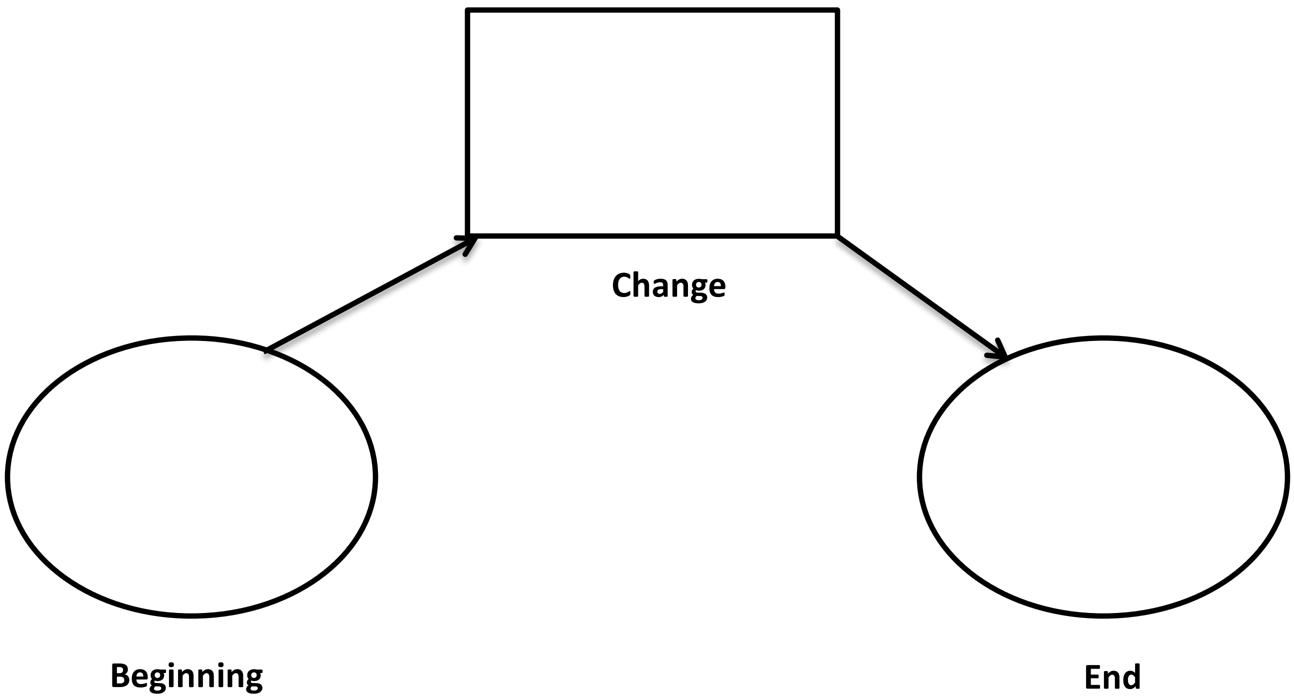


Number sentence:____

Solution:____

Prove your solution is correct in the space below.

Kira ate $\frac{1}{5}$ of her bag of Halloween candy. She has $\frac{4}{5}$ of her bag of candy remaining. How much of the bag of candy did she begin with?



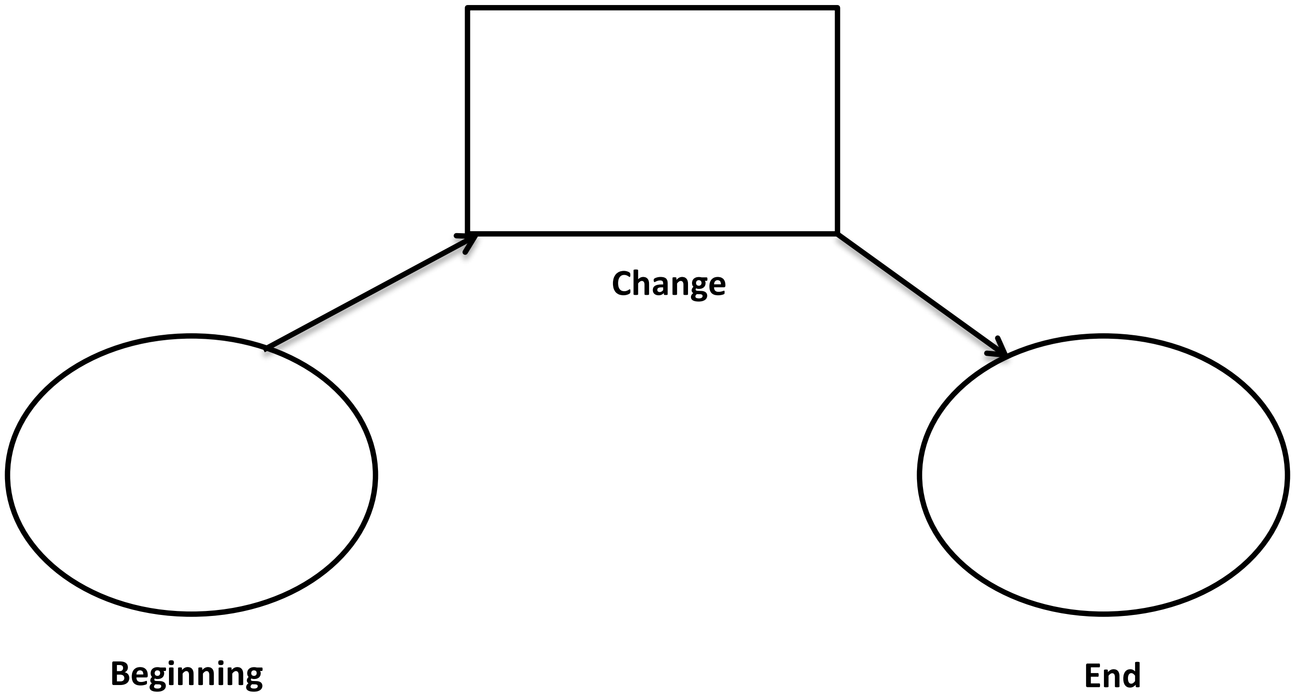
Number sentence:____

Solution:____

Prove your solution is correct in the space below.

Just for Me – Change Story

DeShawn baked brownies for his baseball team party. Before the party started, his little sister ate $\frac{2}{8}$ of the brownies. How much of the brownies does DeShawn now have for his party?



Number sentence:____

Solution:____

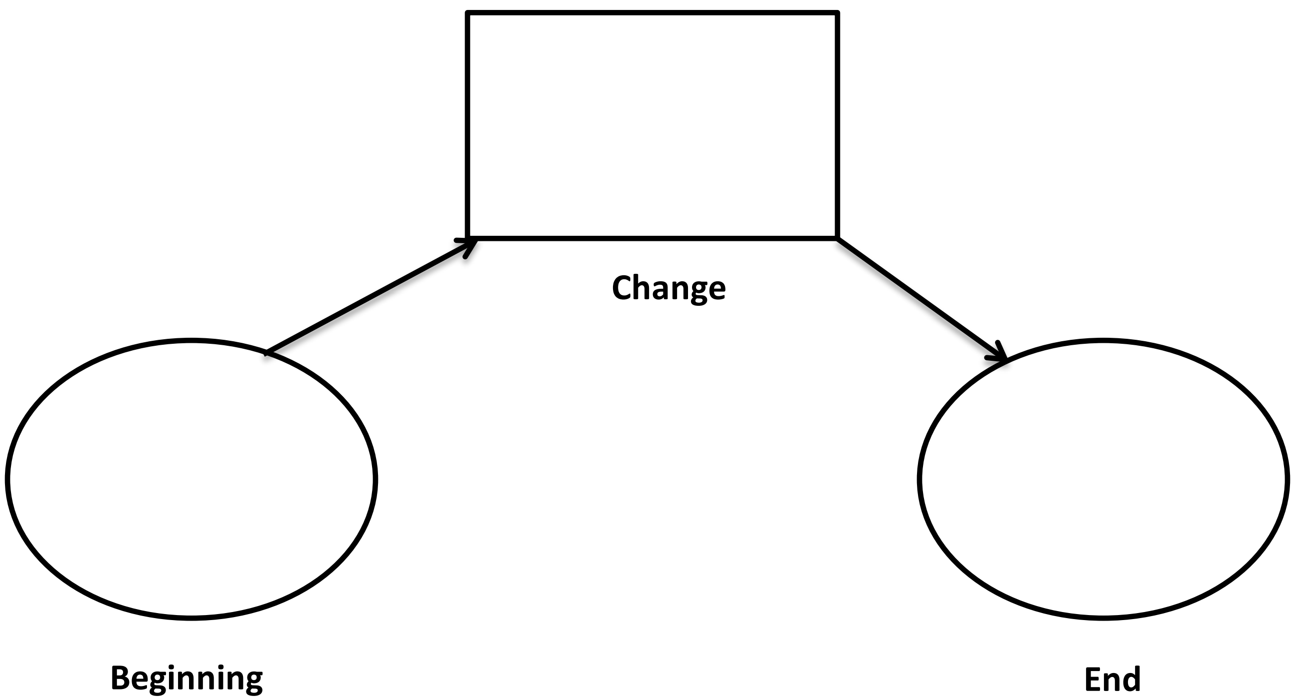
Prove your solution is correct in the space below.

Exit Slip

Name:____

Directions: Solve the problem by using the FOPS strategy and the diagram.

Liam has a Milky Way candy bar. His brother, Nigel, took a big bite of it. Liam now has only $\frac{5}{8}$ of the Milky Way. How much did Nigel eat?



Number sentence:____

Solution:____

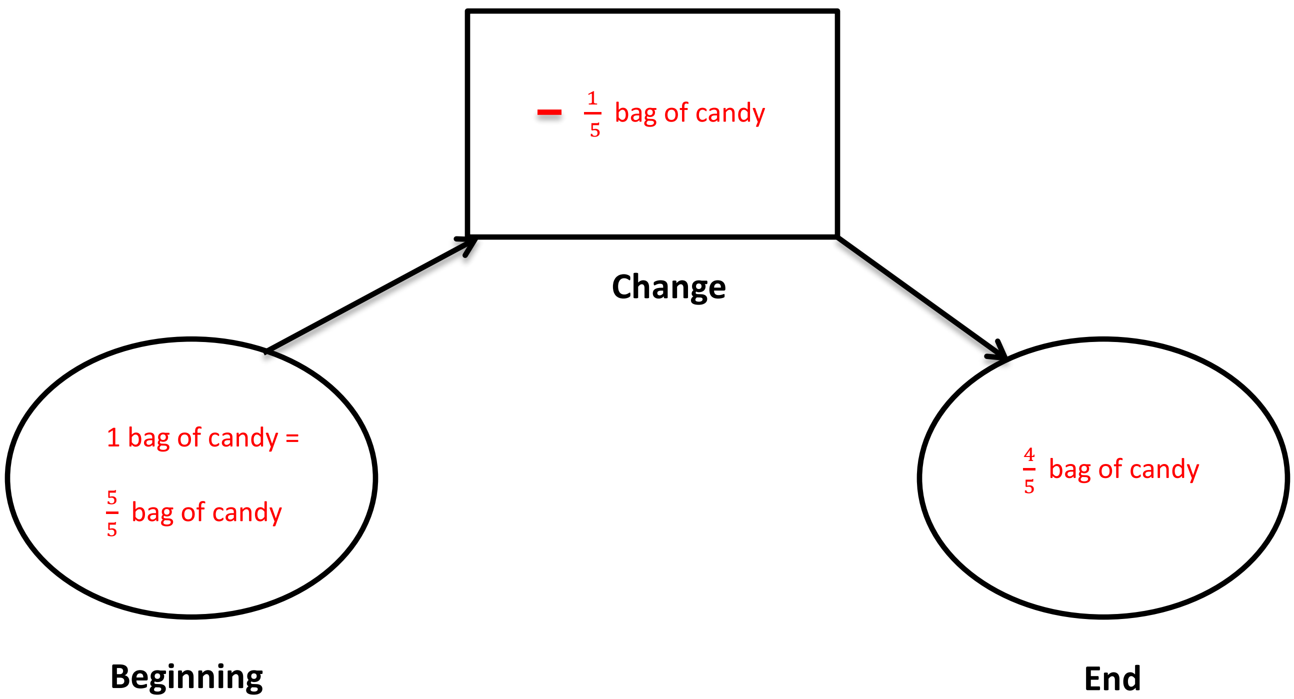
Prove your solution is correct in the space below.

Change Story Problem – Answer Key

Name:____

Teacher Presentation:

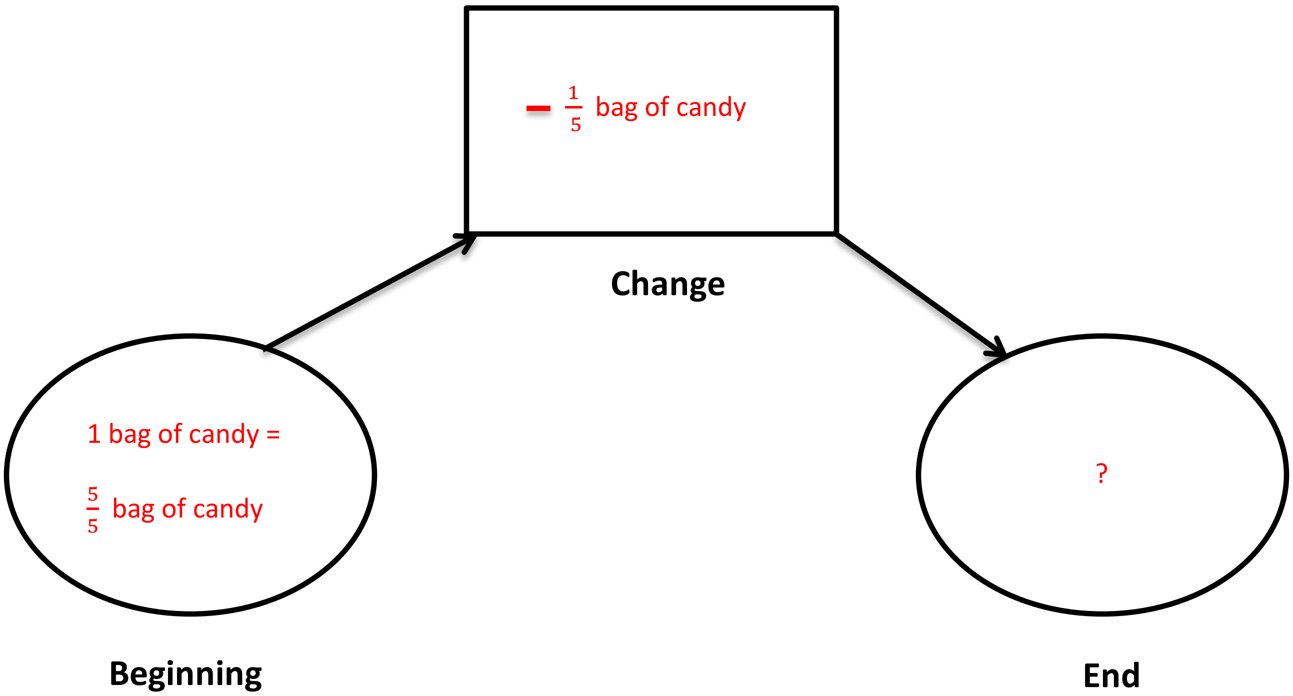
Story: Kira received one bag of candy for Halloween. She ate $\frac{1}{5}$ of her bag of candy. Kira now has $\frac{4}{5}$ of her bag of candy.



Number sentence:___________________ $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$ ____________________

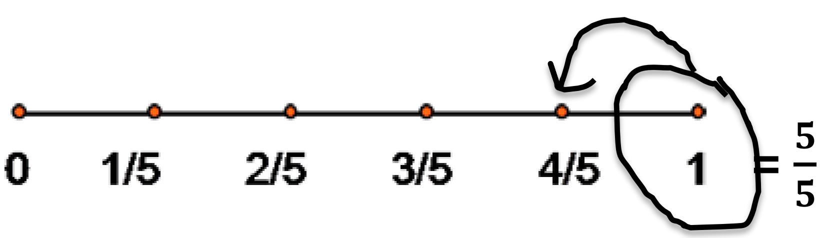
Math Problem Completed during Teacher Presentation

Kira received one bag of candy for Halloween. She ate $\frac{1}{5}$ of her bag of candy. How much of the bag of candy does she now have?



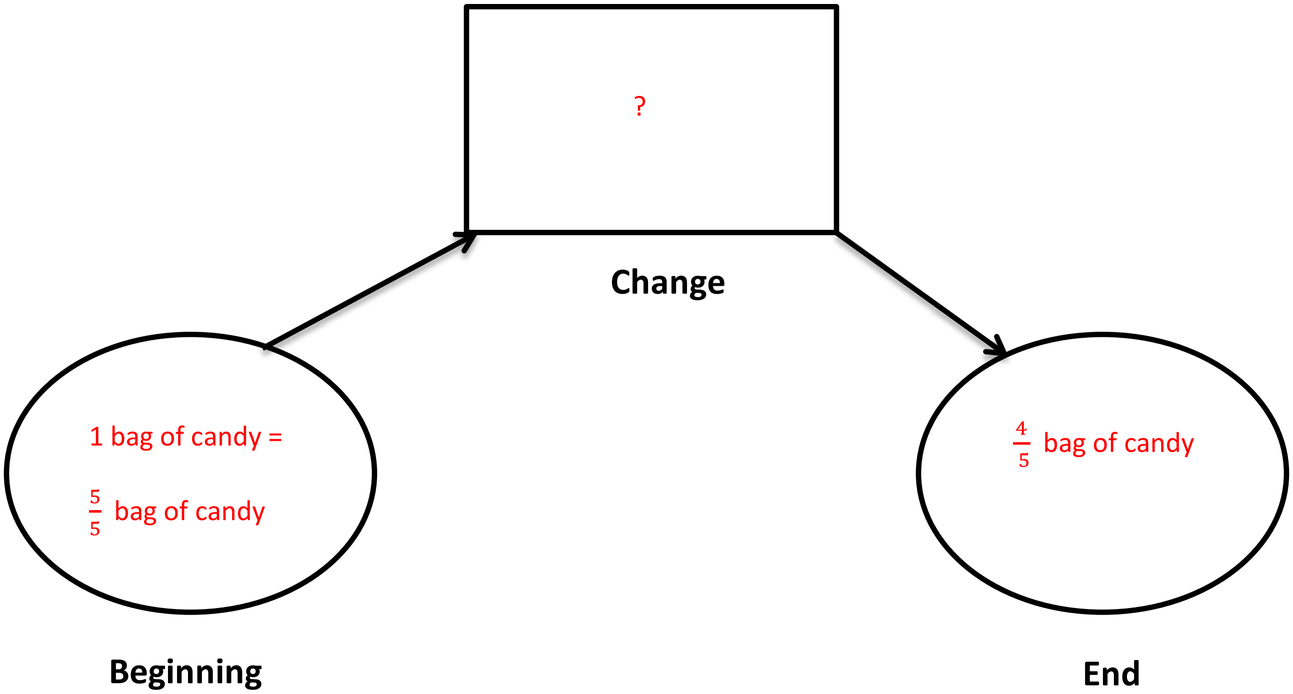
Number sentence:_________________ $\frac{5}{5} - \frac{1}{5} = ?$
Answer: $\frac{4}{5}$ bag of candy

Prove your solution is correct in the space below.



Math Problems completed during Learning Together

Kira received one bag of candy for Halloween. She has $\frac{4}{5}$ of her bag of candy remaining. How much of the bag of candy did she eat already?

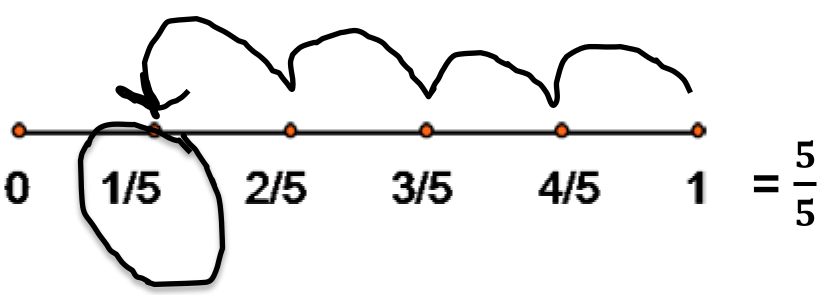


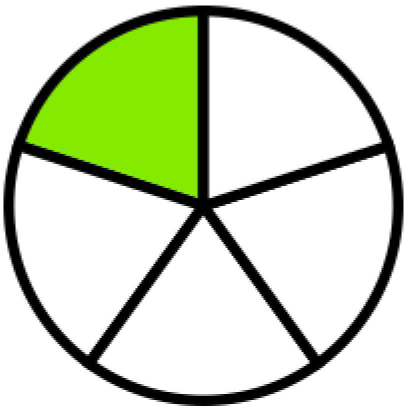
Number sentence:______________ $\frac{5}{5} - ? = \frac{4}{5}$ ___________________________

Answer: $\frac{1}{5}$ bag of candy

Prove your solution is correct in the space below.

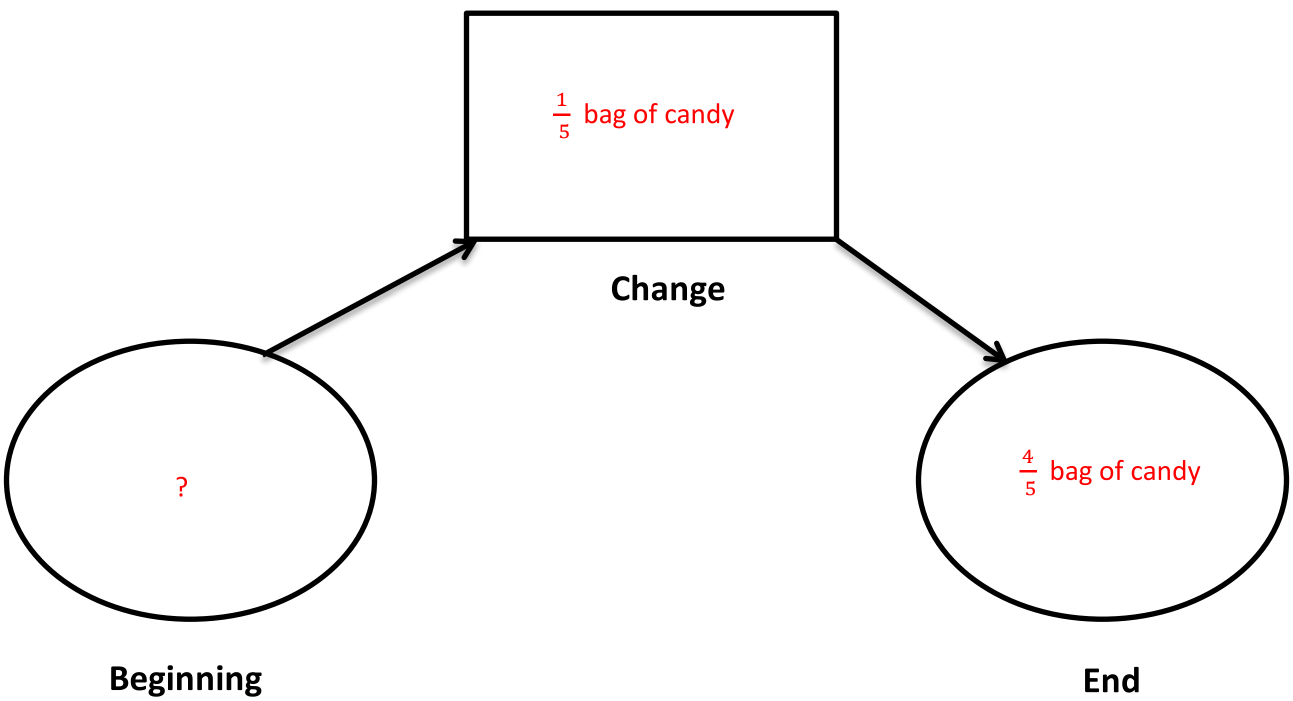
Students may use a number line, manipulatives, or sketches. Or they may explain in words.





1/5 eaten plus 4/5 remaining = the whole bag of candy.

Kira ate $\frac{1}{5}$ of her bag of Halloween candy. She has $\frac{4}{5}$ of her bag of candy remaining. How much of the bag of candy did she begin with?



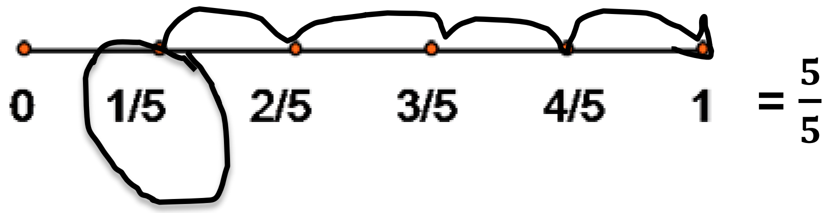
Number sentence:___________? $-\ \frac{1}{5} = \frac{4}{5}$_________

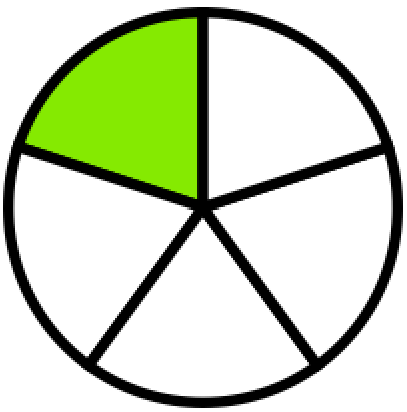
Answer: 5/5 or 1 whole bage of candy

Prove your solution is correct in the space below.

Students may use a number line, manipulatives, or sketches. Or they may explain in words.

Candy eaten plus candy remaining = the beginning value.

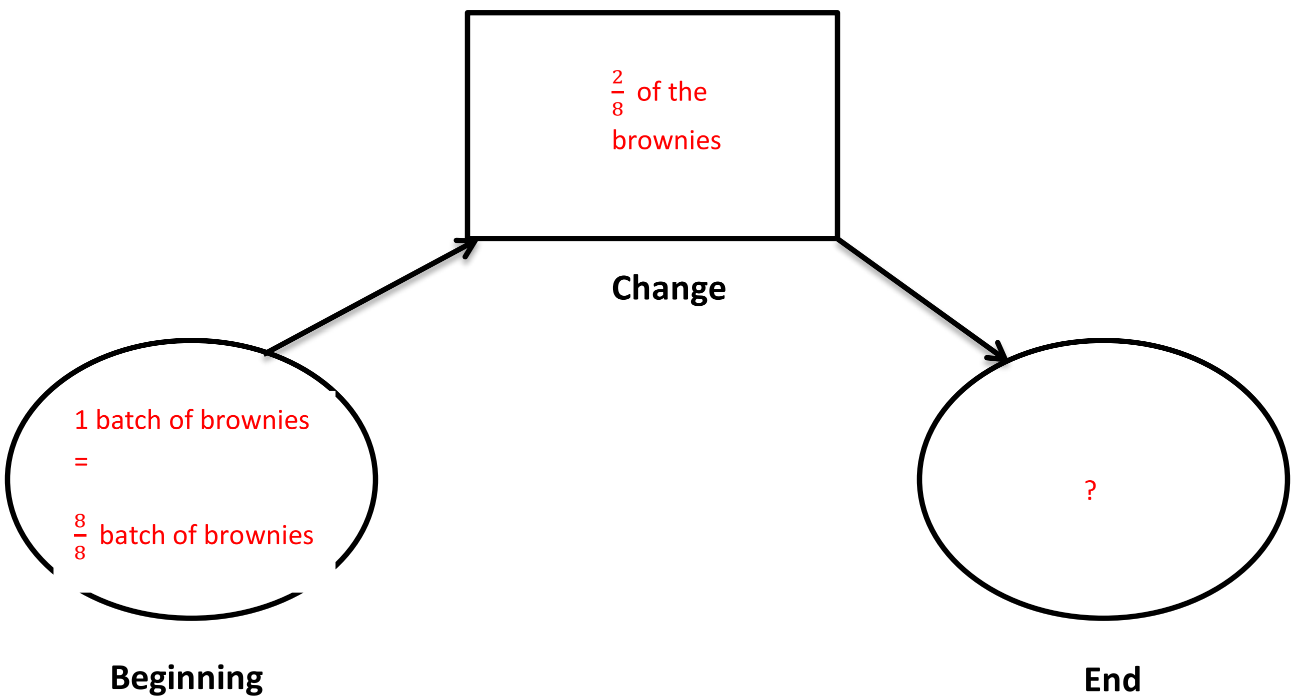




1/5 eaten plus 4/5 remaining = the whole bag of candy.

Just for Me – Change Story

DeShawn baked brownies for his baseball team party. Before the party started, his little sister ate $\frac{2}{8}$ of the brownies. How much of the brownies does DeShawn now have for his party?



Number sentence:________$\frac{8}{8} - \frac{2}{8} = ?$________________________________

Solution:______$\frac{6}{8}$__of the brownies __________________________________

Prove your solution is correct in the space below.

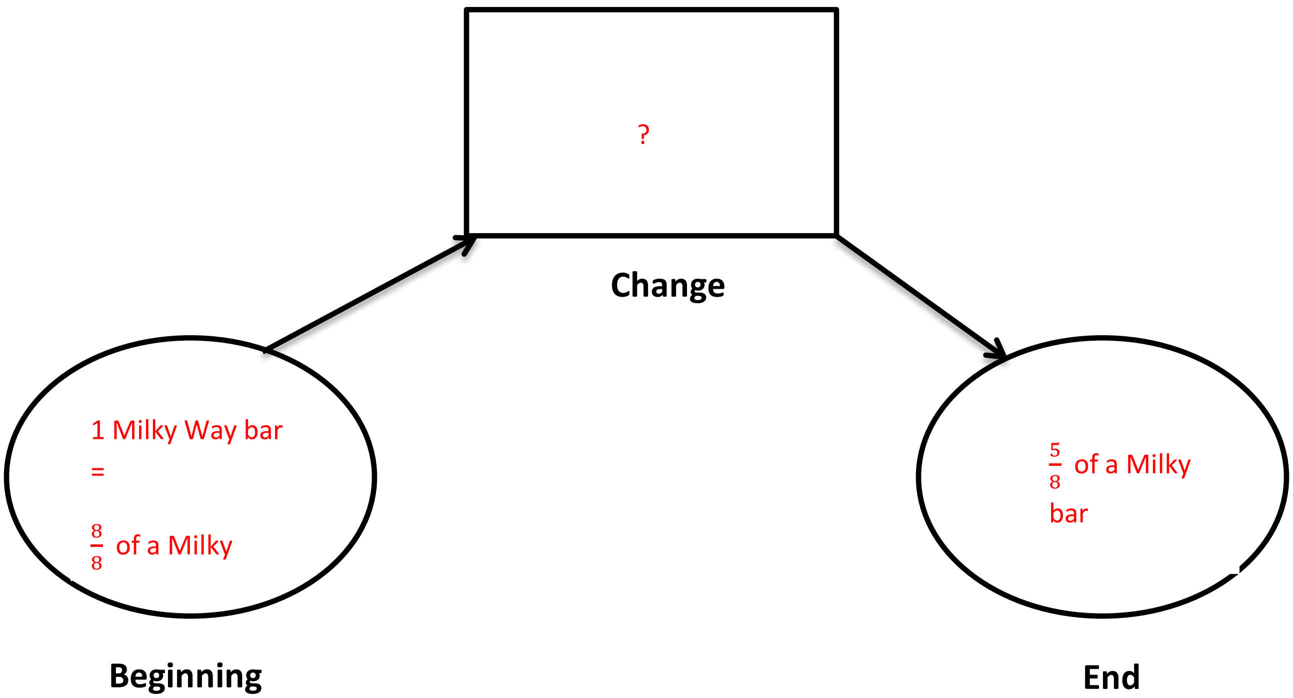
Students may use number lines, manipulatives, or sketches. Or they may explain in words.

Exit Slip

Name:____

Directions: Solve the problem by using the FOPS strategy and the diagram.

Liam has a Milky Way candy bar. His brother, Nigel, took a big bite of it. Liam now has only $\frac{5}{8}$ of the Milky Way. How much did Nigel eat?



Number sentence:___________ $\frac{8}{8} - ? = \frac{5}{8}$

Solution:_______ $\frac{3}{8}$ __of a Milky Way bar _____

Prove your solution is correct in the space below.

Students may use number lines, manipulatives, or sketches. Or they may explain in words.

The Maryland Council of Teachers of Mathematics (MCTM) is pleased to announce that the 2015 Annual Mathematics Conference will be held on October 16, 2015 at Baltimore Polytechnic Institute, 1400 W Cold Spring Lane, Baltimore, MD 21209. This year's theme is "A Walk in the PARCC: Pathways to Success."

For Maryland teachers evaluated with Danielson, attending a professional conference could be an artifact for Components 4d: Participating the Professional Community and 4e: Growing and Developing Professionally.

Conference Program Information:
The 2015 Annual Conference program will be available in September.

Conference Registration Information:
Online registration will be open soon. Onsite registration will be available on the day of the conference. We hope you can join us at the conference!

The Maryland Council of Teachers of Mathematics (MCTM) is the professional organization for Maryland's teachers of mathematics. Our members represent all levels of mathematics educators, from preschool through college. We are an affiliate of the National Council of Teachers of Mathematics (NCTM). Our goal is to support teachers in their professional endeavors and help them to become agents of change in mathematics education.

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City: ________________________ State: ________ Zip: _____________________

Home Phone: ________________ E-mail address: __________________________

MCTM does not share email addresses

School System or Affiliation: ____

Level: Check applicable categories:

__ Early Childhood (Pre-Kindergarten to Grade 2)
__ Elementary (Grades 3 – 5)
__ Middle School (Grades 6 – 8)
__ High School (Grades 9 – 12)
__ Higher Education (Grade 13+)

Signature: _______________________________________ Date:_____________

Join on-line at http://www.marylandmath.org or mail a $15 check, made out to MCTM, and this completed form to:

Dr. Luis Lima
Anne Arundel County Public Schools
Office of Mathematics
2644 Riva Road
Annapolis, MD 21401

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The Maryland Council of Teachers of Mathematics is an affiliate of the National Council of Teachers of Mathematics. Membership in the MCTM is open to all persons with an interest in mathematics education in the state of Maryland. To become an MCTM member, please visit our website: https://www.marylandmath.org/membership/join .

Furthermore, the MCTM Board invites all members to become actively involved in our organization. To become involved, please contact one of the officers listed above. We would love to hear from you!

Maryland Council of Teachers of Mathematics
c/o Sue Vohrer
4 Sheepfold Lane
Hunt Valley, MD 21230

MCTM Mission Statement: The MCTM is a public voice of mathematics education, inspiring vision, providing leadership, offering professional development, and supporting equitable mathematics learning of the highest quality for all students.