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Articles

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REVIEWERS

The individuals below have given their time and expertise to read and review manuscripts submitted for this edition of the *Banneker Banner*. We are very grateful for their help.

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Banneker Banner Submission Guidelines

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. To submit an article, please attach a Microsoft Word document to an email addressed to strickland@hood.edu with "Banneker Banner Article Submission" in the subject line. Manuscripts should be original and may not be previously published or under review with other publications. However, published manuscripts may be submitted with written permission from the previous publisher. Manuscripts should be double-spaced, 12 point Times New Roman font, and a maximum of 8 pages. APA format should be used throughout the manuscript with references listed at the end. Figures, tables, and graphs should be embedded in the manuscript. As the Banner uses a blind review process, no author identification should appear on manuscripts. Please include a cover letter containing author(s) name(s) and contact information as well as a statement regarding the originality of the work and that the manuscript is not currently under review elsewhere (unless accompanied by permission from previous publisher). If electronic submission is not possible, please contact the editor to make other arrangements. You will receive confirmation of receipt of your article within a few days, and will hear about the status of your article as soon as possible. Articles are sent out to other mathematics educators for anonymous review, and this process often takes several months. If you have questions about the status of your article during this time, please feel free to contact the editor. Please note that photographs of students require signed releases to be published; if your article is accepted, a copy of the release will be sent to you and it will be your responsibility to get the appropriate signatures. If you would like a copy of this form at an earlier time, please contact the editor.

Message from the President

Andrew Bleichfeld

Enjoy this Fall 2014 edition of *The Banneker Banner*, the official journal of the Maryland Council of Teachers of Mathematics. Its purpose is to provide professional development and the latest research for the members of MCTM. I hope it is useful to you.

Great thanks go to the Editor of *The Banner*, Tricia Strickland. She has organized for you an issue that contains articles that concern all of us in this time of great change in education. The Editor of *The Banner*, as are all of the MCTM Board positions, is a volunteer position. All the effort expended to bring you *The Banner*, the MCTM Newsletter, the Annual Conference, the Eastern Shore Conference, and the future Dine and Discuss sessions is being done by volunteers. We hope you appreciate all of our efforts and judge MCTM to be one of your favorite resources. Please look for those Dine and Discuss sessions in the next few months. They are opportunities for quick and purposeful professional development sessions simply for the benefit of our members. There will be twelve sessions held around the state. Each will be a quick two-hour session held on a weeknight. Hopefully there will be one held in your area. Sessions are already being scheduled for Frederick County, Howard County, Harford County, and Baltimore City. Look for announcement of the sessions on our website, www.marylandmath.org. We hope you can attend one or more, and can bring a few of your colleagues with you to enjoy some snacks and some great professional development.

As always, your MCTM Board and I are here to serve you. Please contact us at any time to let us know what we can do for you.

CSA/CRA in the Classroom

By Nicole Shilling
Frederick County Public Schools

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

Starting with the graduating class of 2018, students will need to take a math class each year of high school. This new requirement is intended to increase the rigors of high school mathematics and to ensure that Maryland students are ready for college level mathematics while possessing career readiness math skills. This requirement raises concerns for students that already struggle in the area of mathematics as well as the educators that are entrusted with their education.

Rigor in the classroom is intended to raise the level of work so it is challenging and students are able to display a deeper mathematical understanding of math. The students' knowledge is evaluated through high stakes testing such as the High School Assessments (H.S.A.). The Algebra I H.S.A. is given near the end of the course and the results are reported to Maryland State Department of Education. The results found that 84.4% of all 10th graders, 88.7% of all 11th graders and 88.3% of all 12th graders who took the Algebra I H.S.A. passed; however, only 47% of all 10th graders, 54.9% of all 11th graders and 60.2% of all 12th graders with an Individualized Education Program (IEP) passed (Maryland State Department of Education, 2013). Based on these results, students with a disability appear to be having difficulties in mathematics. Students with a math learning disability (MLD) often have trouble communicating their understanding of math, reading directions and problems, writing answers and understanding verbal directions (Strickland, 2014). This makes learning mathematics difficult and leads to frustration and ultimately to math anxiety. However, there are instructional strategies to help all students learn mathematics.

CRA Instruction

A concrete-semi-concrete-abstract (CSA), also known as concrete-representational-abstract (CRA), instructional strategy is a sequence that supports the basic understanding before applying he math rules to problems (Gagnon & Maccini, 2001). The first stage is concrete where the students have physical objects in their hands which can be manipulated by the students. This is known as the "doing stage" (The Access Center, 2004, p.1). The semi-concrete or representational stage, students are taking the concrete items and making it representational on their paper. This is known as the "seeing stage" because the students are "drawing pictures; using circles, dotes" or "tallies" (The Access Center, 2004, p.1). The abstract stage is using mathematical symbols only to solve problems without any help from physical objects nor drawings. This is the stage where students are doing math in their heads, completing math problems that are written down where the students have to use paper and pencil to solve the problems (MathVids, 2014, para. 34). The CRA sequence blends together both "conceptual and procedural understanding" of mathematics (University of Kansas, 2014, para. 11). The graduated instructional sequence of CRA is when the students master each level first before moving on compared to the integrated instructional sequence of CRA which allows the students to see the connection of all three stages right away.

Strickland and Maccini (2012) applied CRA-Integration strategy (CRA-I) into a study with three boys who were taking Algebra 1 or Pre-Algebra and also had learning disabilities. The CRA-I strategy takes the basic idea of CSA/CRA and modifies it. Instead of mastering one stage before moving onto the next stage, CRA-I strategy has the students complete all three stages at once. Firs the student will use the concrete materials, then draw it on paper, and then write a mathematical representation of it. In Strickland and Maccini (2012) research, they found that using the CRA-I strategy and expansion boxes it was possible for students with learning disabilities to learn how to multiply polynomials (Strickland & Maccini, 2012). Strickland and Maccini (2013) continued their research in a study with five participants who all had a history of math difficulties and were at risk for failing their Algebra 2 class. All participants had improvements to their algebraic achievements and continued success with the expansion boxes (Strickland & Maccini, 2013). These studies showed how CRA-I strategy help students with learning disabilities understand Algebra easier and fuller. The CRA-I strategy allows students to feel the concrete, learn how to draw the concrete on paper, and then how to write it mathematically. This process allows students to see how all three connect together and make connections to the real world. The purpose of this study is to see if the CRA-I strategy helped students in a co-taught class understand the probability unit easier and to have a deeper understanding of probability.

Method

The participants in the study needed additional instructional strategies to help them understand and retain information taught in Algebra I. The CRA-I strategy seemed to fit with the amount of time the participants had for the study.

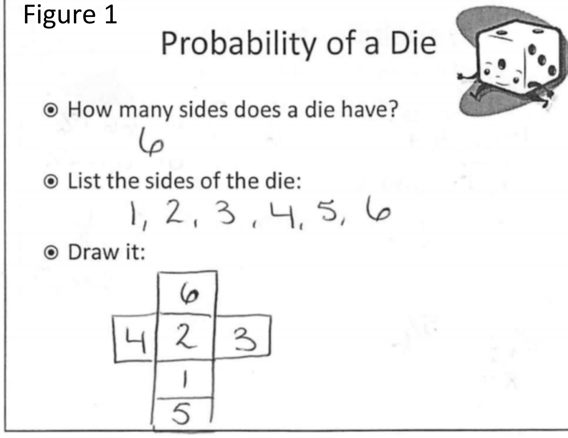
Participants and Setting

The study took place in a public high school located in Frederick County, Maryland. All twenty-one participants were enrolled in a co-taught Algebra I course which consisted of six girls and fifteen boys. From this group, one participant had a 504 Plan and seventeen participants had an IEP. Out of the seventeen participants, eight participants were identified as having a specific learning disability (LD) as their primary disability on their IEPs, four participants were identifies as other health impairment (OHI), four participants had autism, and one participant's primary disability was not identified at time of the study. Out of the eighteen students who had a 504 Plan or IEP, ten participants had each assignment modified for them, as part of their accommodations. Modified assignments means that there are less words on the page, sentences and paragraphs are broken down to where the students can understand the reading, there are fewer problems for the students to complete and in the case of multiple choice, the number of choices is also reduced for example from four choices to three choices.

Measures

Quantitative data was collected through a pre-test and a post-test. The pre-test consisted of questions about simple probability, independent probability, dependent probability, mutually exclusive events and inclusive events. The pre-test was given before probability was introduced. The post-test consisted of the same questions as the pre-test and was given at the end of the probability unit.   Qualitative data was collected through a survey which was created in Google Forms and filled out by the participants during class time.

Figure 1



Procedure

At the start of the study, the participants took a pre-test to show what they already know about probability. After the pre-tests were graded, the researcher had a better understanding of what the students knew and did not know. First, the researcher focused on just simple probability with a single die (Figure 1). The participants first got to hold a die and look at all the sides. The researcher then asked the participants how many sides are on the die and what is the likely hood would be if the class rolled the die that the outcome would be six. Once the participants understood the physical, the researcher had the students draw the die on a piece of paper so that the participants could see a pictorial representation of the object. Once the participants understood about a single die, the participants worked in pairs to determine all the possible outcomes of rolling a pair of dice. Finally the participants drew a table to show all the possible outcomes.

Secondly, the participants held a coin in their hands, drew it on paper, and answered probability questions about it (Figure 2). Thirdly, the participants held a standard deck of cards. The participants had to count out the number of cards in the deck and record the information. Then the participants had to separate and count based on color (red or black), suit (hearts, diamonds, spades, or clubs) and face value (Ace, 2, 3, 4, …, King). The researcher then asked the participants about simple probability of a standard deck of cards. For example, what was the probability of getting a red card? The participants could either say the answer from their memory, look at their notes or physically pull out the red cards and count them.

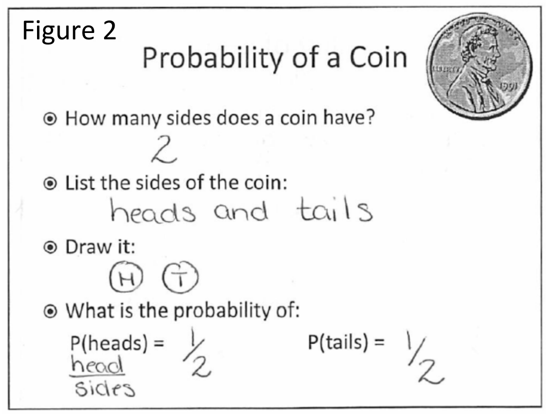


Figure 2

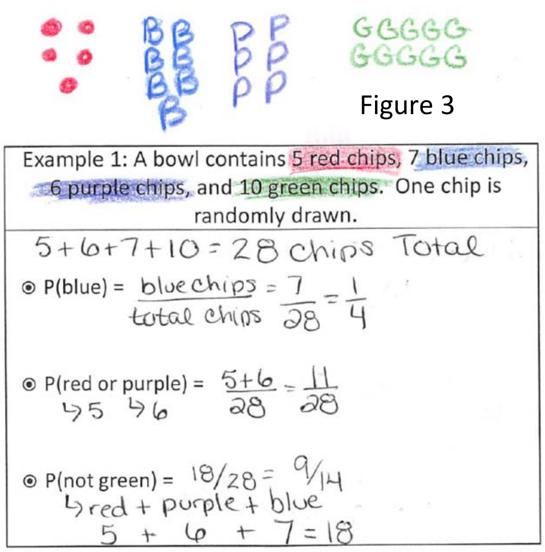
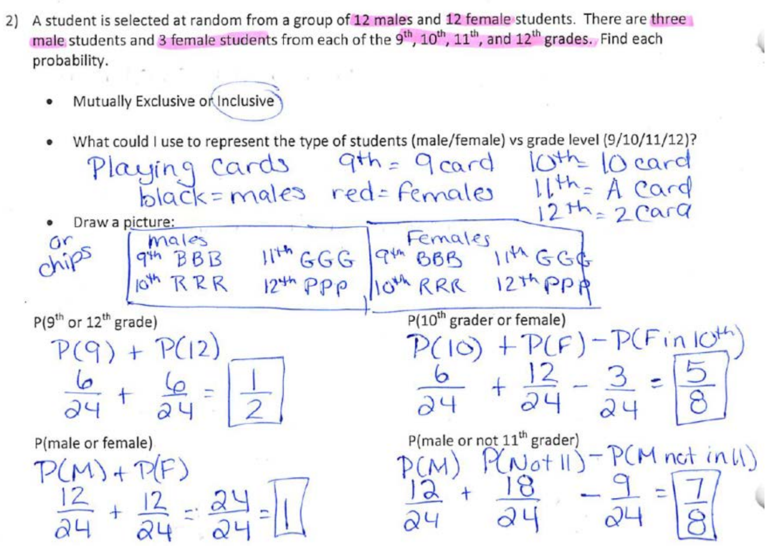


Figure 3

Finally, the participants learned simple probability from clear blue, green, red and purple chips (Figure 3). Each participant had fifteen chips of each color. In order to answer any probability question on their worksheet, the participants had to have their chips in front of them as well as drawn on their paper and the math to answer the question. This allowed the students to find the simple probability of each color.

Figure 4



2) A student is selected at random from a group of 12 males and 12 female students. There are three male students and 3 female students from each of the 9th, 10th, 11th, and 12th grades. Find each probability.

- Mutually Exclusive or Inclusive
- What could I use to represent the type of students (male/female) vs grade level (9/10/11/12)?

Playing Cards 9th = 9 card 10th = 10 card
black = males red = females 11th = A card 12th = 2 card

- Draw a picture:

or chips

Males: 9th BBB 11th GGG 10th RRR 12th PPP
Females: 9th BBB 11th GGG 10th RRR 12th PPP

P(9th or 12th grade)
P(9) + P(12)
$\frac{6}{24} + \frac{6}{24} = \boxed{\frac{1}{2}}$

P(10th grader or female)
P(10) + P(F) − P(F in 10th)
$\frac{6}{24} + \frac{12}{24} - \frac{3}{24} = \boxed{\frac{5}{8}}$

P(male or female)
P(M) + P(F)
$\frac{12}{24} + \frac{12}{24} = \frac{24}{24} = \boxed{1}$

P(male or not 11th grader)
P(M) P(Not 11) − P(M not in 11)
$\frac{12}{24} + \frac{18}{24} - \frac{9}{24} = \boxed{\frac{7}{8}}$

The participants mastered simple probability, the research moved onto finding the probability of independent and dependent events, and then mutually exclusive versus inclusive events (Figure 4). During these lessons, the participants would pull out their concrete materials, draw it on their paper and answer the question using just the math. The last two days of the unit were review days for the participants to make sure they full understood the material by completing a study guide which offered more questions than what was going to be on their post-test. The post-test was exactly the same as the pre-test with the exception that one of the colors got change. Two school days after the post-test, the participants were asked to fill out a survey asking how they felt about using hands on material in class, did they feel like it helped them and would they like to use them again.

Results

Results in the study show that using the CRA-I strategy helped the participants learn about probability. From the initial twenty-one participants, twenty of them completed the study, one participant decided to stop coming to school. Nineteen of the twenty participants showed improvement from their pre-test to post-test.

The pre-test and post-test was graded out of thirty-five points and converted into percentage for comparison. Pre-test scores averaged 11.65% while post-test scores averaged 40.65%, giving an increase of 29% points. The lowest pre-test and post-test scores were 3% and 11% respectively. The highest pre-test and post-test scores were 29% and 86%, respectively. Every participant scored an F on the pre-test but only seventeen participants scored an F on the post-test. Two participants scored a D and one participant scored a B. Eighteen participants raised their post-test score by at least 10% points, one participant raised their post-test by 3% points and one participant scored the exact same score on their post-test.

Figure 5

	Mean Pre-Test	Mean Post-Test	Mean Increased by
Whole Class	11.65	40.65	29
IEP	11.125	40.88	29.75
Non-IEP	13.75	39.75	26
LD	13.48	45	31.58
Austim	6.25	34	27.75
OHI	9	29.25	20.25
Mod	10.2	32.1	21.9
Non-Mod	13.1	49.2	36.1
Boy	8.8	37	28.2
Girl	12.6	41.87	29.27

The whole class was broken down into male vs female, IEP vs No IEP, type disability, mod vs non-mod (see Figure 5). Male participants had a pre-test scores averaged 8.8%, post-test scores average 37%, an increase of 28.2% points, while female participants had a pre-test scores average 12.6% and post-test scores average 41.87, an increase of 29.27% points. The participants who have an IEP had a pre-test scores averaged 11.125%, post-test scores averaged 40.88, an increase of 29.75% points. The participants who did not have an IEP had a pre-test scores averaged 13.75%, post-test scores averaged 39.75%, an increase of 26% points. The participants who have a LD had pre-test scores averaged 13.48%, post-test scores averaged 45%, an increase of 31.58% points. The participants who have autism had pre-test scores averaged 6.25%, post-test scores averaged 34%, an increase of 27.75% points. The participants who have an OHI had pre-test scores averaged 9%, post-test scores averaged 29.25%, an increase of 36.1% points. The participants who receive their assignments modified had a pre-test scores averaged 10.2%, post-test scores averaged 32.1%, an increase of 21.9%. The participants who receive no assignments modified has a pre-test scores averaged 13.15%, post-test scores averaged 49.2%, an increase of 36.1% points. Each group increased their mean post-test scores by at least 20% points. Six of the groups, male, female, IEP, Non-IEP, LD, and autism, all came within the same 3% point increase that the whole class. Two groups, OHI and MOD, came in 7.1% point and 8.75% point, lower than the whole class increase. One group, non-MODS, came in 7.1% point above the whole class increase.

Participants also had to fill out a survey consisting of three questions. The first question was: How much did you like using the hands on materials? One participant loved using the hands on material, fourteen liked using the hands on material, five participants neither liked nor disliked them and no participants disliked or hated them. The second was: Do you feel like using hands on materials help you understand the material easier? Two participants said no that it did not help them. One participant said no because it took too much time. Eight participants said that it helped them because it was easy. Nine participants said that it was helpful because it was fun and filled their interests. The materials were hands on, they could see how many items there where, which then allowed them to see the problem better or what was in the problem. Since the participants had the material, they could spread the items out and get the answer. One participant even said that it helped them out a lot and they did not have to think a lot.

The third was: Would you like to use more of these types of items in your math class? One participant said no, another said no because they already have a lot to do and a third said that it does not matter to them. Two participants said yes and fifteen others gave a reason. One participant would like to use the hands on material as long as it does not take up a lot of time. Others thought it made the class more fun and helped them with their work. Some participant liked the activity plus it made learning the material easier. One participant even had a warning, to watch out for the ones who play with the materials. Overall, 75% of the participants loved or liked the materials, 85% felt like it helped them learn the material and 85% would like to use similar materials in other math classes.

Discussions

The purpose of this study was to see if the CRA-I strategy would help students in Algebra to understand and retain the information taught to help them pass the course, pass the Algebra I High School Assessment and to be successful in society. The study did show that CSA-I strategy helped students learn and remember the material easier. Eighteen of the twenty participants increased their post-test scores by at least 10% points.

The CSA-I strategy allowed the participants to make the connections between the real world, the representational world and the abstract math world to fully understand the unit being taught. This can be seen when the participants are broken down into groups. The fact that most of the groups are within 3% points increase difference shows that this instructional strategy really helped most of the participants. This showed that there is about one or two participants that needed additional instructional strategies to help them be successful in Algebra I.

The participants brought up good points in the survey about using the concrete materials. First, students need to be taught how to use them otherwise the lesson will be lost on the students. Secondly, teachers need to allow more concrete materials in the classroom to help the students fully understand the material. Finally, not all students will need the hands on material, which means that multiple worksheets need to be created for students to move about the levels of CSA on their own. Nineteen participants found some success in this study.

Limitations and Future Research

During this study it is important to note that this group of participants was a unique group. Frederick County Public Schools is in the process of transitioning from the old curriculum to the new Common Core curriculum. The new curriculum must be in place by the 2014-2015 school year, it was decided to get rid of a course called Intro to Algebra and place the students who would normally go into that class for freshmen year, go directly into Algebra I Yearlong. Students who normally go into Algebra I Yearlong, got put into Algebra I 18 week. This created a co-taught class with 85% of the class with an IEP compared to other years where it is a maximum of 50%. Since there is a high number of participants with IEPs the data collected in this study could be skewed. A future study would be take two or three classes that are more balanced and see if the same results come up.

Another limitation that came up during this study is that the participants needed a lot help. The participants had a tough time working independently; the participants needed constant help and step by step directions of what to do. Another future study is to create worksheets that would walk the participants through each stage of CSA. This way the participants are getting consistent practice of what to do first, second and third. This might allow the participants to be able to do it on their own once no support is given. These worksheets can made in a range of ways such as ones with all three stages, ones with only the last two stages, ones with just the last stage, others with no stages. This would also allow participants work at their own pace and master the curriculum when they fully understand it.

A third limitation is when the study took place. The study took place in late April to early May, which is when seniors get senioritis and underclassmen are done with school. The participants were tired of school work, completing math work and wanted the summer to be there already. A future study could take this into account and conduct a study in the middle of the school year or at the middle of the semester. Both of these options would allow for students to be engaged in the lessons being taught. With taking into consideration these three limitations, a new study can be produced to be the best study of CSA/CRA in the classroom.

Practical Implications

This study showed how much students in high school like using concrete materials. It is not only fun for them but also makes the learning the material easier for them. This should not be a surprise to any teachers but it is. Students start learning in elementary school with a lot of hands on materials. In middle school, students start get less hands on materials and by the time the students get to high school they have very few opportunities with hands on materials. Some of this has to do with the undergraduate training that teachers get but it also stems from the fact that most secondary math teachers do not think of math as abstract in the Algebra 1, Geometry and Algebra 2 because the teachers fully understand it. This research has shown that teachers need to find more ways to bring the real-world into the classroom, and that does not mean more word problems. This might mean bringing in a real ladder so students can see how a ladder leans against a house or how a ramp is shaped for a trigonometry problem. This is where collaboration between teachers becomes important, teachers need to share what works so that other children can be reached. There are many instructional strategies out there, teachers just have to take the time to search them out.

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Success for Students with Learning Disabilities in the Algebra Classroom

Brittany Beresford

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

Success for Students with Learning Disabilities in the Algebra Classroom

I am a high school mathematics teacher, and I just completed my third year at a high school in Frederick County, Maryland. Although I have become a better teacher since August 2011, I still find many ways to improve my teaching. Knowing that I can always do better helps me to grow and therefore helps my students as well.

This past year, one of our school-wide goals showed me a way that I could improve my teaching in a big way. My principal asked us to target students that were eligible to receive special education services and students that were eligible for Free and Reduced Price Meals (FARMs) for our Student Learning Objectives (SLO's). Although intuitively I realized that these students underperformed compared to their peers, I had never looked at the evidence that supported this claim.

I want all of my students to do well, and I know that something needs to change. The students that I researched for this paper are those with learning disabilities (LD). Learning disabilities contain the following: dyslexia (processing language), dyscalculia (math skills), dysgraphia (written expression), dyspraxia (fine motor skills), auditory processing disorder, visual processing disorder, and Attention-Deficit/Hyperactivity Disorder (National Center for Learning Disabilities [NCLD], 2014). Because of the high possibility of having students with LD in our inclusive classrooms, I wanted to research more about this group of students.

Current Maryland Math Graduation Requirements

In Maryland, students must fulfill several math requirements. Current high school students must earn three math credits (Maryland State Department of Education, 2013). They are required to take Algebra and Geometry and then may choose one additional credit (Maryland State Department of Education, 2013). In addition, there is a new law for students that will enter ninth grade in the 2014-2015 school year which requires students to enroll in a math class every year until they graduate (Maryland Public Schools, n.d.).

In addition to earning these math credits, students must pass standardized assessments, such as the HSA Algebra assessment or the new PARCC Algebra 1 assessment (Maryland State Department of Education, 2014). Furthermore, although students do not have to take Algebra 2 in order to graduate, they must pass an assessment at the end of eleventh grade; the PARCC Algebra 2 can be used for this (Maryland State Department of Education, 2014).

In summation, beginning in the 2014-2015 school year, in order to graduate, ***all*** current ninth grade students must complete one math class each year, pass the PARCC Algebra 1 assessment, and pass a second assessment.

Support against Requiring Algebra for All Students

Now that I have discussed the math graduation requirements, I want to discuss whether or not we should have these requirements for all students. I first want to go over research about why students should not be forced to take certain classes, especially Algebra. The research in this section does not specifically mention students in special education, but it discusses exempting some students from certain classes. Therefore, I believe that we can apply this information to our students in special education.

Chazan (1996) stated that students from working-class backgrounds probably do not see their parents using algebra, and therefore they do not see a need for learning it. Chazan argued that motivating unwilling students with the Algebra curriculum was a virtually impossible task because it was difficult to show how they would transfer those skills to their daily lives. Chazan wanted to see a change in the Algebra content so that students could see how they would use it in their lives and why they must take it. I believe that the Common Core standards are moving towards Chazan's vision because it requires lessons to have real world examples and more problem-solving than memorizing.

When we cannot motivate our students, they do not put in the necessary effort in order to understand the material. This can lead to failure, with students that must repeatedly take the course while remaining uninterested and becoming even more frustrated. Morgatto (2008) states, "Students with difficulties in mathematics may be frustrated with algebra and denied the opportunity to learn other mathematical concepts that are more relevant to their needs" (p. 216). Again we see the need for a change so that all students can succeed in a way that benefits them.

Noddings (2007) suggests that one possible change should be individualized educations and states, "When we insist that all children can and must learn academic mathematics, we hurt them doubly. They fail in subjects they have not chosen, and they are deprived of studies at which they might excel" (p. 27). Although I want every student to love math as much as I do, I also do not want to force them into a class that they will struggle through and not profit from. One suggestion is to offer other math classes, depending on what the student plans to do after graduation (Morgatto, 2008). We need to make school a place that students want to come to and give them an education that teaches them content that they will use.

I agree with these authors to an extent. We do not want to limit a child's possibilities, but I do agree that forcing students to learn math that we cannot relate to something outside the classroom is neither necessary nor valuable. Education must offer a positive experience and must also help, not hinder, students' growth and futures.

Support for Requiring Algebra for All Students

Although I see students struggling with math every day, the fact that I am a math teacher makes me want to believe that all students need math and can be successful at it. The following section discusses why Algebra is important for all students, no matter their mathematical ability, needs, or plans after high school.

One major reason for requiring all students to take Algebra is that we want all students to have the same opportunities (Morgatto, 2008). Impecoven-Lind and Foegen (2010) state, "Algebra is considered a gateway to expanded opportunities for students of all races and cultures (Fennell, 2008), facilitating achievement in advanced mathematics courses, entrance into college, and economic equity in the workforce" (p. 32). School leaders have determined graduation requirements (which includes Algebra), so that we do not limit what a child can do after high school.

Another argument is, no matter what a student does after high school, he or she will need to have math knowledge. When taught correctly, students develop strategies for coming up with a solution, persevering, and supporting their answer. These skills can then be transferred to solving other real-life problems. According to Basham and Marino (2013), "In many countries, including the United States, careers requiring an applied understanding of STEM [Science, Technology, Engineering, and Math] are quickly replacing traditional manufacturing jobs" (p. 8). Many people argue that low achieving math students will just get jobs that do not require math, but because of the increase in jobs that require math, this would lower their job-acquiring potential. Looking at the trends in the United States and around the world, the National Science Foundation determines what students should learn in school (National Science Foundation, 2012). This has caused schools to require more rigorous science and math classes. We want to prepare all students for whatever they decide to do.

I like this side of the argument because I want to believe in my students. I never want to tell a student that they cannot or should not take a class. Instead, I want to develop my classes so that they fit students' needs and contribute to students' futures. If implemented correctly, I believe that the Common Core standards will benefit our students' futures and increase their academic achievement in math.

Ways to Support Students with Learning Disabilities in Math

In order to increase student achievement, however, we must take extra steps with our students in special education. Under the Individuals with Disabilities Education Improvement Act (IDEIA), teachers must know the information in a student's Individualized Education Plan (IEP) and provide appropriate services for the student in the classroom (Yell et al., 2008). Because it is a requirement for all students to take and pass certain math classes, we must figure out how to help them succeed. This section discusses research about teaching students with learning disabilities and ensuring that we meet their needs.

"The best way to meet the needs of this group of students, as well as other students, is for teachers to understand and apply the principles of Universal Design for Learning (UDL)" (McLaughlin, 2012, p. 23). Universal Design for Learning requires that teachers present information in multiple ways, allow students to respond in multiple ways, and inspire student interest in the topics ("About UDL", n.d.). With this method, students are given the choice of which tools they want to use in order to solve the problem. Our students want to be engaged while learning, and using multiple ways for students to connect to the material and incorporating technology will increase their engagement.

Technology also benefits students with LD by providing accommodations. These students have a learning disability that prevents them from performing at the same level as their peers. This is why students' IEPs state accommodations that they require in order to help them achieve. A common way to provide these accommodations is to use technology. Bouck and Flanagan (2009) state, "Assistive technology (e.g., calculators and computer-assisted instruction) may be a means of providing assistance by increasing access to mathematical ideas and helping them experience higher levels of success" (p. 18). Sometimes we force students to solve long problems by hand that a calculator or computer can quickly solve for us, even though determining what the answer means or how we can use that answer is the most valuable part of the process. Of course doing things by hand is sometimes necessary and beneficial, but providing assistive technology can eliminate unnecessary struggling and allow all students to increase their performance.

In terms of lesson structure, many researchers suggest that one of the best ways to teach students with learning disabilities is by using explicit instruction (Gersten et al., 2009; Kiuhara & Witzel, 2014; McLaughlin, 2012; Scheuermann, Deshler, & Schumaker, 2009; Strickland & Maccini, 2010; Wilson, 2013). Strickland and Maccini (2010) define this method, "Explicit instruction is a method of teacher-directed instruction that incorporates the following teaching functions: an advanced organizer, teacher demonstration, guided practice, independent practice, cumulative practice, and curriculum-based assessments to provide data to drive instructional planning" (p. 39). Furthermore, Gersten et al. (2009) analyzed 11 studies that used explicit instruction and found that it significantly increased student learning. By allowing the students to first see the teacher modeling the skills and then practicing together, students can gain confidence in their mathematics ability.

In addition to using explicit instruction, teachers of students with learning disabilities should also use the concrete-representational-abstract (CRA) approach (Gersten et al., 2009; Strickland & Maccini, 2010; Witzel, Riccomini, & Schneider, 2008; Witzel, Smith, & Brownell, 2001). In the concrete stage, students use counters, blocks, algebra tiles, or geoboards; in the representational stage, they use drawings, pictures, and virtual manipulatives (Strickland & Maccini, 2010). The CRA approach helps students to understand and generalize in the abstract stage because they have already mastered the concrete and representational stages. This also allows for multiple opportunities for students to acquire a skill.

Providing multiple opportunities ensures that students have been given enough situations to become proficient in understanding the concepts (Rivera & Baker, 2013). Repetition and connections will help students to remember what they have learned and to help them learn new content. In addition, because students in special education tend to have lower mathematics literacy with daily life math problems, these skills should also be addressed while working through the curriculum (Rivera & Baker, 2013).

Additionally, Common Core stresses using real-world examples, and using enhanced anchored instruction (EAI) incorporates these real-world connections. According to Strickland and Maccini (2010), "EAI is specifically designed to improve the mathematics and problem-solving performance of secondary students with LD and involves the use of video-based problems and hands-on activities with group activities" (p. 42). It is also helpful for students to discuss their reasoning (Kiuhara & Witzel, 2014). After researching student inquiry, Gersten et al. (2009) stated, “One very promising finding is that the process of encouraging students to verbalize their thinking or their strategies, or even the explicit strategies modeled by the teacher, was always effective” (p. 57). Enhanced anchored instruction uses real life examples in order to encourage problem solving and group work and has been shown to benefit students with learning disabilities.

When planning lessons, teachers need to think about their students in special education. I believe that doing this will benefit all students and will help the teacher develop creative, engaging lessons.

Conclusion

Math class requirements and the math curriculum are big issues. This is especially true for what we expect of students with learning disabilities. When talking about education for all, does this mean that every student must meet the same requirements or should every student receive an individualized education based on their skill levels and needs? Many arguments exist for both sides of this debate, and I believe that both sides have note-worthy points.

Some people believe that each student should receive an education based on what they will do after high school and what they are willing and able to handle. Obviously students have their own strengths, and schools should not punish a student because they do not have advanced skills in math. People that do not want Algebra as a graduation requirement see individualized educations as a way to help all students succeed in their own way.

People on the other side of this debate are also concerned with students’ futures. They believe that requiring higher-level math classes is necessary and will only serve to advance our students. We now live in a technologically-advanced world and the skills learned in school must reflect that. Increasing the rigor in the math classes will also help students in other aspects of their lives. People that want Algebra as a graduation requirement believe that everyone needs those skills to succeed, both in their careers and in their daily lives.

Math is a very important skill that students need to learn. If we can create math lessons that correlate to the students' lives, then we have a better argument for requiring all students to take the class.

I hope that the Common Core standards are the necessary adjustment needed in order to help all students to succeed. However, in order to make this possible, teachers must be willing to plan appropriate lessons and provide necessary accommodations to students in special education. When teachers have knowledge of and have prepared for their students' learning needs, success for all students in algebra is possible.

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Mind the (Math Gender) Gap

Jamie Geiser

Research reported in this paper was completed in a capstone project for the master's degree in mathematics education in the Graduate School of Hood College.

Some claim that males are the superior gender. Males are the physically stronger gender, which led them to have an advantage from the start. In the beginning males were the hunters and the protectors. They were the go-getters, and became the leaders. At one point, men had more rights than women and a higher education was one of those rights. On the other hand, maternity was the basic role of women, which resulted in a stereotype that a woman's place was in the home; and therefore, they only received enough education to fulfill that role (National Women's History Museum [NWHM], 2007). In the past, and even sometimes currently, men have been considered to be better at math than women. Some people even think that males are hardwired to perform better at math. One piece of evidence to support the claim that males are better at math than females is the difference between their scores on standardized math tests. In this article, I will examine the gap in performance on the SAT and mathematics AP test scores and I will also provide some research on why this gap currently exists.

History of Educating Women in Math

Colonial Education. Formal education for girls historically has been secondary to that for boys (NWHM, 2007). In the 1700's neither colonial American families nor their teachers expected girls to prepare for a future other than that of marriage and motherhood; therefore, a middle-class girl learned from her mother's example that cooking, cleaning, and caring for children was the behavior expected of her when she grew up (NWHM, 2007). With that in mind, girls only learned enough mathematics to enable them to figure out the household bills while a boy's education prepared them to take over the family business or to become a thriving participant in society by studying higher math and other areas like Greek, Latin, science, celestial navigation, geography, history, fencing, and plantation management (Stratford Hall Plantation, n.d.). This educational trend has been changing in recent decades, and girls are taking over jobs that were once thought to be men's work.

Sophie Germain. Despite the lack of support, some women still managed to prevail in mathematics. Sophie Germain (April 1, 1776 - June 27, 1831), known for making important contributions to the areas of number theory and mathematical physics, began teaching herself mathematics using the books in her father's library. Like most people from her time, her parents felt that her interest in math was inappropriate for a lady, and they did everything they could to discourage her attempts to learn mathematics. Sophie resisted this idea and began secretly studying math at night. When her parents found out, they went to such extremes as taking her clothes away at night and depriving her of heat and light in an attempt to make her stay in bed. This, of course, did not stop Sophie. She found ways to combat her parents by camouflaging herself in quilts and using concealed candles in order to study clandestinely. Finally, her parents gave in after realizing that they could not take away Sophie's passion for mathematics (Swift, 1995).

Sonya Kovalevskaya. Another woman who beat the odds of her time was Sonya Kovalevskaya (January 15, 1850 - February 10, 1891). Sonya was an influential mathematician, writer, and advocate of women's rights in the nineteenth century. She began studying differential and integral calculus at the age of eleven by reading her father's old calculus notes that wallpapered her nursery. In 1874, Sonya became the first woman to be granted a Ph.D. in mathematics when she earned her doctorate from Göttingen University (Wilson, 1995). Today, many colleges and universities in the United States hold annual Sonya Kovalevskaya Math Days for high school girls to celebrate mathematics and to encourage girls studying mathematics (Association for Women in Mathematics, 2010). These women and their perseverance paved the way for women in mathematics today, and their efforts do not go unnoticed.

Experiments

Endorsing Stereotypes. A study led by Sian Beilock (2009), a psychologist at The University of Chicago, looked at how math anxiety of female elementary schoolteachers in the United States has an impact on the achievement of their female students. First and second grade teachers completed an evaluation on their own math anxiety. The math achievement of the students in these teachers' classrooms was also assessed. At the beginning of the school year, there was no relationship between a teacher's math anxiety and her students' math achievement. However, by the end of the school year, the female students' performance began to reflect the anxieties of their female teachers thus enhancing the stereotype that boys are good at math, and girls are good at reading. Girls who endorsed this stereotype were less successful in math than girls who did not (Beilock, Gunderson, Ramirez & Levine, 2009).

Stereotype Threat. Other studies show that the difference in math performance between men and women could be eliminated when the stereotype is deemphasized. One particular study tested this idea by randomly sampling thirty women and twenty-four men from the introductory psychology participant pool at the University of Michigan. Two tests were administered to each subject who were given fifteen minutes to complete each test. Half of the subjects were told that the first test was one in which there were gender differences and that the second test was one in which there were no gender differences, and the other half was told the exact opposite and then the subjects were then randomly assigned to these order conditions. (Spencer, Steele & Quinn, 1999). The instructions went on to say that the first test had been shown to produce gender differences and that the second test had been shown to produce no such differences, or vice versa, depending on the order condition. When participants were explicitly told that the test yielded gender differences, women greatly underperformed in relation to men. However, when the test was claimed not to yield gender differences, women performed at the same level as equally qualified men. This happened, of course, even though the test in these two groups was the same (Spencer et al., 1999). Therefore, one way to address women's underrepresentation in mathematical pursuits is to create environments without this phenomenon known as *stereotype threat* (Pronin, Steele, & Ross, 2004). This type of environment can be created through positive mentoring in which female students are assured they do have the ability to succeed.

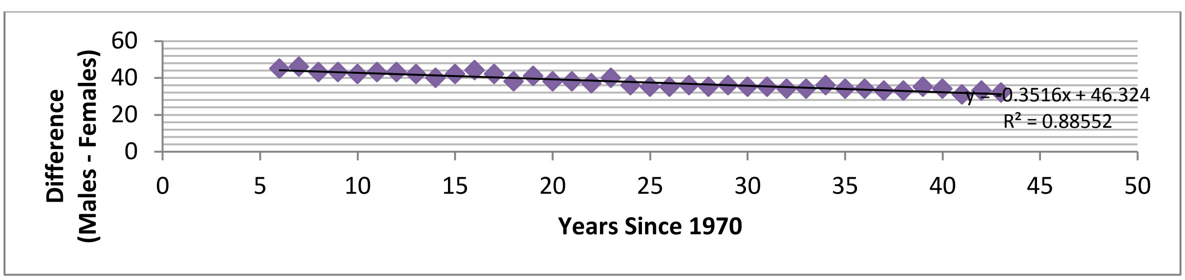
The Solution

Programs. With better role models and more opportunities to engage in mathematical activities, young girls can overcome the stereotype and close the math achievement gap. There are programs such as the Sonya Kovalevskaya Math Days that encourage young girls in math. Another such program is Girls to the Fourth Power. This program, funded by Tom Peters, is committed to overcoming the "math block" that is widely perceived to affect many girls (Meehan, n.d.). Their goals are to improve math skills and increase self-esteem and self-confidence in young women (Meehan, n.d.). Expanding Your Horizons Network (EYHN) is another program that was created to inspire girls to recognize their potential and pursue opportunities in STEM. This program is a one-day conference for girls that includes hands-on STEM activities and engages girls with female STEM role models. EYHN is known for being girls' gateway to STEM. The program is designed to spark a girl's imagination and transform her image of what is possible and who she can be to include STEM (EYHN, 2013).

The Data

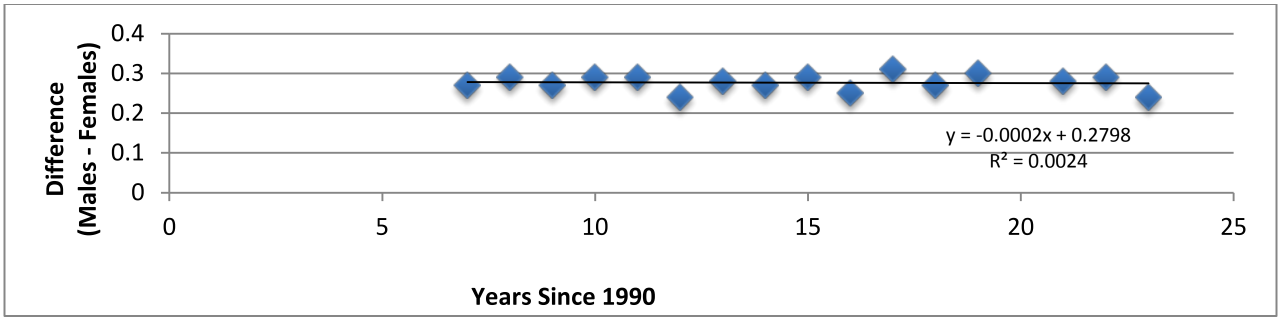
Most articles to date on the math achievement gap between males and females focus on the significant difference between their performances. I think a more important focus should be whether the gap is closing or not. We want to see that over time girls' achievement will hopefully one day catch up to that of boys, closing the gap permanently. In an investigation to see if the math achievement gap is closing, I decided to focus on the math Scholastic Assessment Test (SAT-M) and AB Calculus, BC Calculus, and Statistics Advanced Placement (AP) test scores.

After obtaining both the average SAT-M scores and AP scores for males and females for, I calculated the average difference in scores between males and females (males – females). I performed a linear regression t-test for the difference between the average male and average female scores over time in hope to see that the true slope, β, is negative. A negative slope would mean that over time the average difference between male and female scores is decreasing. The scatterplots are shown below.



t = -16.687 df = 36 p = 0

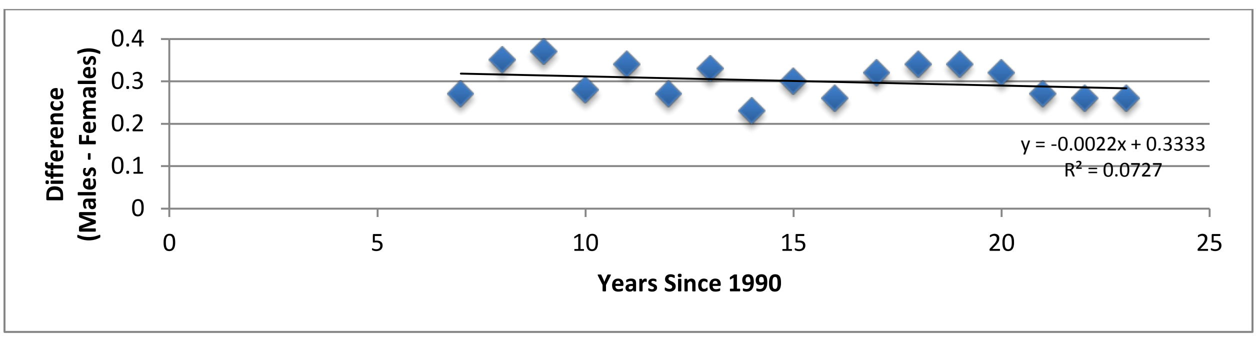
Figure 1. Linear Regression t-Test for the Average Difference in SAT-M Scores Between Males and Females Over Time.



t = -0.18507 p = 0.427915 df = 14 s = 0.020923 r = -0.0494

Fail to reject H_o at $\alpha = 0.05$

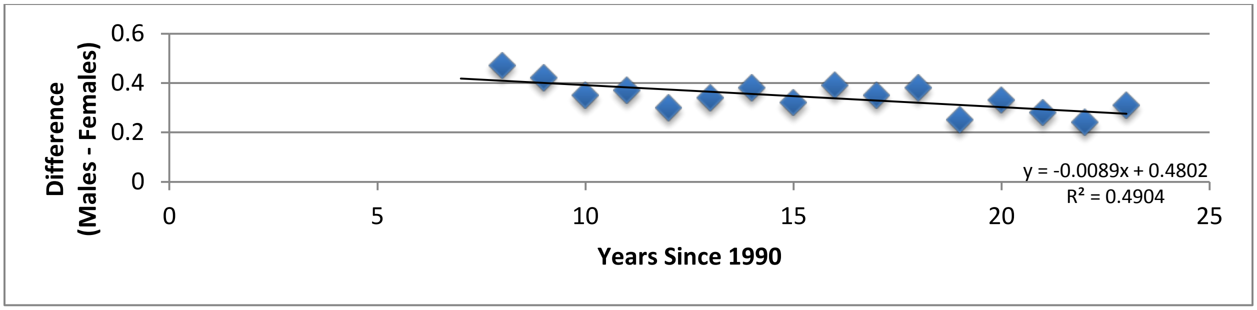
Figure 2. Linear Regression t-Test for the Average Difference in AB Calculus Scores Between Males and Females Over Time.



t = -1.08466 p = 0.147599 df = 15 s = 0.040622 r = -0.26968

Fail to reject H_o at $\alpha = 0.05$

Figure 3. Linear Regression t-test for the Average Difference in BC Calculus Scores Between Males and Females Over Time.



t = -3.6704 p = 0.00126 df = 14 s = 0.04462 r = -0.70028

Reject H_o at $\alpha = 0.05$

Figure 4. Linear Regresttion t-Test for the Average Difference in Statistics Scores Between Males and Females Over Time.

In my linear regression t-tests for β I used the null hypothesis H_o: $\beta = 0$ and the alternative hypothesis H_a: $\beta < 0$, where β is the true slope of the regression line comparing year and the average difference in SAT-M scores or AP scores.

With such a small p-value for the SAT-M we are able to reject H_o at $\alpha = 0.05$. The data provide convincing evidence that girls are in fact closing the math achievement gap. Even though males are still performing better than females in math, the gap is indeed closing and will continue to close as long as we make conscious efforts to eliminate stereotypes, and to provide equal opportunities for both genders. The only AP test that shows a significant decrease in the difference between the average male score and average female score over time using $\alpha = 0.05$ is the Statistics test. While the other two tests have slight negative slopes, the slopes are not negative enough to be considered significant.

Conclusion

Change takes time and we cannot undo many years of history over one school year. What we can do is focus on ways to promote the change we desire to see. By placing emphasis on encouraging girls to be successful in algebra from the start, we can close the gap simply by nurturing girls and their inherent mathematical abilities. If these girls become elementary school teachers, their female students will likely be more confident in mathematics. This creates a circle of events that will most certainly close the existing math achievement gap between boys and girls. While boys are currently outperforming girls in math, the achievement gap is steadily closing, and will continue to close as long as we take it upon ourselves to negate the stereotypes that currently exist.

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Interactive Resources for the Teaching of Common Core High School Geometry

Luis Lima

The 2014-15 academic year marks the first full implementation of Maryland's College and Career Ready Standards (MCCRS) (MSDE, 2003a). Based on the Common Core State Standards, the MCCRS embody hopes of improved educational outcomes for thousands of Maryland students. These standards hold an opportunity for education equity "by ensuring all students are taught to the same high standards and held to the same rigorous expectations" (Center for American Progress, 2014).

> It's a seismic shift in education meant to better prepare kids for college, career and the global economy. But new standards as rigorous as the Core require lots of other changes – to textbooks, lesson plans, homework assignments. In short: curriculum and the materials needed to reach it. And that's the problem. Right now, much of that stuff just isn't ready. (Turner, 2014)

Despite the dearth in curriculum and materials, teachers will still have to design and deliver instruction that advances the mastery of both content and practice standards in support of students' college and career readiness (Gojak, 2013).

If teaching to new standard expectations without aligned materials were not challenging enough, the teaching of Euclidean geometry, which is the focus of most high school geometry courses, presents its own challenges. It is not unusual for students to fail to reach the deductive reasoning level expected by the end of the course, with many students showing minimal progress in reasoning levels - as measured by the van Hiele model (van Hiele, 1999; Usiskin, 1982). This issue may also be compounded, in part, by the fact that novice geometry students struggle with a certain "inability to reconcile the chasm between *intuition* and the *formalism* in the prevailing presentations of the subject" (Wu, 2013, p. 154).

Regarding the presentation of high school geometry, research suggests that the use of "[d]ynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena"(NGACCSSO, 2010). Similarly, Whiteley (2004) postulates that "visual representation and thinking skills can be as important to students' futures as the symbolic and language based reasoning" (p. 1). The use of dynamic software seems to offer teachers and students the possibility of constructing and analyzing mathematical relationships in terms of loci that result from moving elements within the representation of the problem (Santos-Trigo, 2008). Moreover, Clemments (2003) also observed that

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According to the van Hieles, the learner, assisted by appropriate instructional experiences, passes through the following five levels of geometric thought:

Visual Level – the student identifies, names, compares and operates on geometric figures according to their appearance.

Descriptive Level – the student analyzes figures in terms of their components and relationships among components and discovers properties/rules of a class of shapes empirically.

Informal Deduction Level – the student logically interrelates previously discovered properties/rules by giving or following informal arguments.

Formal Deduction Level – the student proves theorems deductively and establishes interrelationships among networks of theorems.

Rigor Level – the student establishes theorems in different postulational systems and analyzes/compare these systems.

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Adapted from Fuys, Gedes, & Tischler (1988)

> [s]tudents believe that it is possible to abstract the properties of geometric objects from diagrams directly and thus deduce a property empirically. Interactive geometry software introduces a new type of diagram whose behavior is controlled by theory. Students' actions require construction of an interpretation in which visualization plays a crucial role but geometrical properties constrain such actions. (p. 159)

Therefore, the intent of this article is to highlight a few free online resources that use animations and visualizations that support the design of robust high school geometry materials (lesson plans, homework assignments, performance tasks, and units of study) to meet the expectations of the MCCRS for Geometry. These online resources, which focus primarily on online manipulatives, could be used to create instructional materials that not only support students' engagement that allow them to progress along the van Hiele levels of geometric thought (CCSSO, 2010; Fuys, Geddes, & Tischler, 1988; van Hiele, 1999; Wu, 2013) but also take advantage of these manipulatives' unique affordances to support students' development and internalization of the mental actions associated with the acquisition of content (Sarama & Clements, 2009).These instructional materials should be introduced as thoughtful sequences of computer activities and teacher mediation of students' work with those activities to build an effective learning environment (Clements, 2003).

Online Resources

1. **Math Open Reference** - http://www.mathopenref.com/index.html

This site holds a collection of illustrations that are animated and interactive and constitutes an online library of digital manipulatives in plane geometry, coordinate geometry, solid geometry, trigonometry, calculus, and other general tools. Teachers and students can manipulate these animations to illustrate a significant number of geometry concepts. Operation is intuitive and in most cases involves moving an orange dot(s) on screen with your mouse. Figure 3 below shows the animation for rotating the yellow figure around center C by dragging the orange dot at the bottom right-hand corner. Since center C is also indicated by an orange dot, students may also explore the effect on the rotation of changing the center of the rotation.

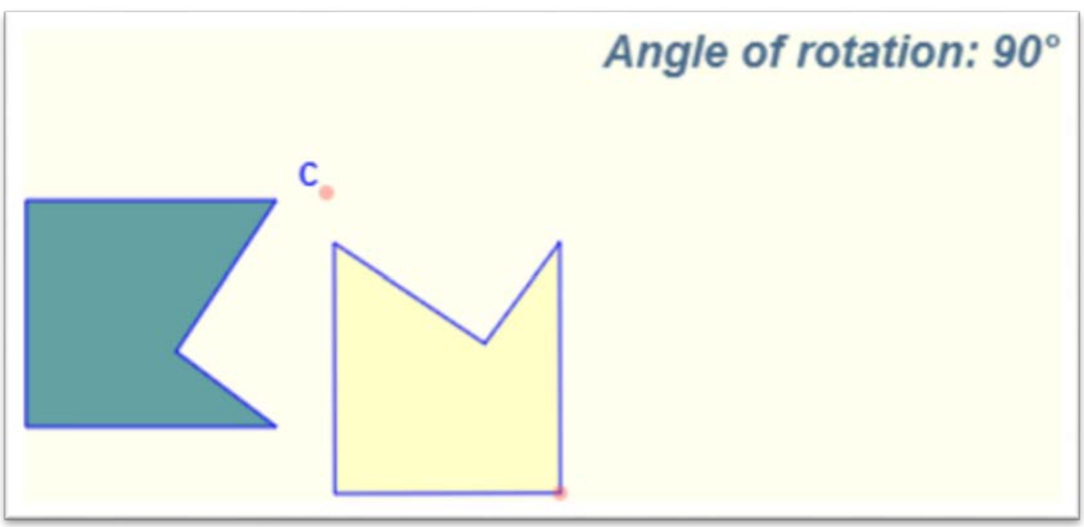

Figure 1 - Rotation about center C

One of the highlights of this collection is the set of accurate, realistic, and animated constructions using a compass, a straightedge, and a pencil. Students can control the viewing of animations so they can either watch the complete construction or freeze-frame through, step-by-step, to develop skill in creating them or use the available worksheets with step-by-step instructions.

Having access to fully-animated and controllable demonstrations affords teachers the opportunity to provide individualized support to students in this key area of the high school geometry course. Students can watch and interact with the animation to develop mastery in construction using a compass, a pencil, and a straightedge.

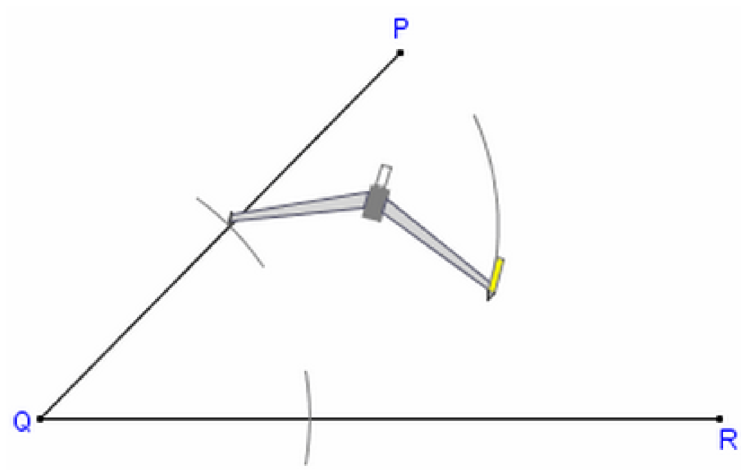

Figure 2 - Animated Euclidian construction: Angle bisection

The ability to make accurate Euclidian constructions using a compass and straightedge is one of the standards emphasized in the CCSSM high school geometry course. Constructions also play an important role in helping students understand multiple axioms and theorems in the course and are used in the PARCC assessment to gauge student understanding of a wide range of theorems and their proofs. The site also correlates the animations with the CCSSM, to facilitate planning of activities that support the curriculum.

2. **Inter*activate*** – Transformations, Reflections, and Rotations

   http://www.shodor.org/interactivate/lessons/TranslationsReflectionsRotations/

The site was developed by the Computational Science Education Reference Desk and licensed by Shodor, it offers teachers a thorough lesson plan including alignment to standards and textbooks, prerequisite skills, a complete outline, student activity sheet and a link to an online applet, called the Transmographer, that helps students make meaning out of transformations.

The Transmographer works with a square (default), parallelogram, and a triangle and allows for translations, rotations, and reflections that are easily controlled by the online commands. On the Transmographer page, tabbed as Activity, teachers can also navigate to additional resources on the tabs (a) Learner, (b) Help, and (c) Instructor.

Under the Learner tab users will be able to find the definition of Transmographer and links to additional resources including other activities, discussion prompts, and worksheets. Under the Help tab, users find a detailed guide on how to manipulate the applet in order to allow students to learn about transformations; whereas under the Instructor tab users will find an abbreviated version of the lesson outline with links to the same set of resources available under the Learner tab.

This site is particularly relevant to high school geometry because it allows students to understand transformations from an analytic geometry standpoint, which directly supports the content standards CCSS-M (G-CO.2 to 6). Additionally, it builds on transformations as one of the foundational concepts of the course since, under the CCSSM, G-CO. 7 to 8; rigid transformations are used to explain triangle congruence in terms of rigid motions in a very concrete and tactile manner.

Another set of activities with the Transmographer can be found at http://www.shodor.org/interactivate/activities/TransmographerTwo/, where you can create your own polygon. This site shares the same features and structures as the previous one.

3. **Inter*activate*** – Cross Section Flyer

http://www.shodor.org/interactivate/activities/CrossSectionFlyer/

This is another one of Shodor's sites dedicated to the exploration of conic sections. It displays the standard features of all Shodor sites: a thorough lesson plan including alignment to standards and textbooks, prerequisite skills, a complete lesson outline, and a student activity sheet. The interactivity is derived from students manipulating different sliders to move the intersection plane through any one of five solids that can serve as a basis for this investigation. Figure 2, below shows how this site could be used to help students visualize relationships between two-dimensional and three-dimensional objects (G-GMD.B-4). Additionally, by selecting a cone as the solid to be sectioned, students can investigate how conic sections are formed and leverage the dual representation (3-D and coordinate plane) to learn to translate between the geometric description and the equation of conic sections(G-GPE.A-1 to -2).

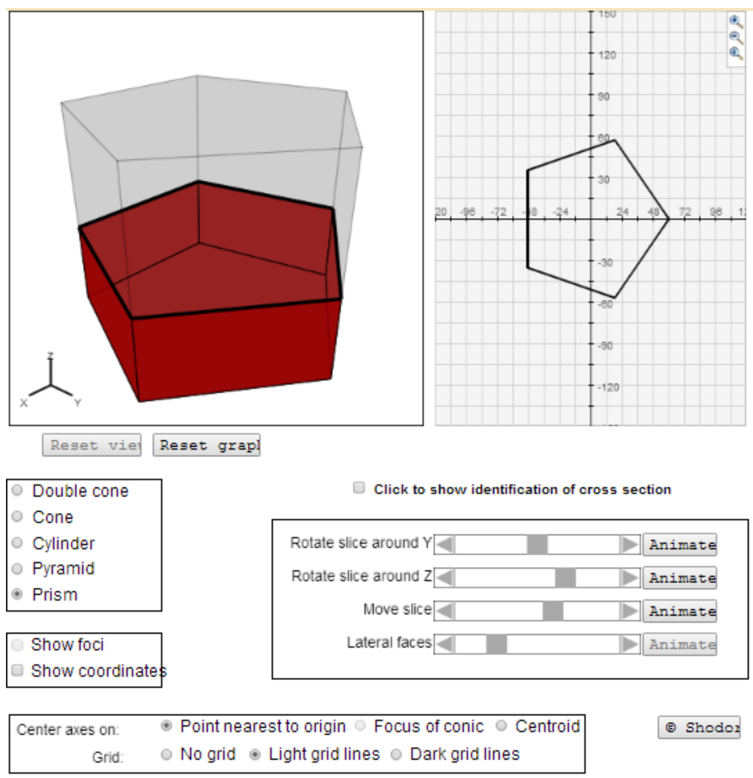

Figure 33 - Conic Section Flyer

National Library of Virtual Manipulatives (9-12 Geometry)http://nlvm.usu.edu/en/nav/category_g_4_t_3.html

This is perhaps the largest collection of online virtual manipulatives designed for grade school mathematics. The segment dedicated to high school geometry contains 30 distinct applets that deal with the visualization of concepts such as fractals and transformations as well as with virtual manipulatives such as geoboards, tangrams, turtle geometry, and tessellations. Of particular interest to this article are the five transformation applets that allow students to visualize and manipulate dilations, reflections, rotation, translation, and compositions in an interactive online environment. The site provides support to multiple languages, including Spanish, French, and Mandarin. Teachers utilizing this online resource will have to option to have students create their own shapes and apply the desired transformation to it. Each transformation page allows the use to toggle the coordinate plane on/off, which allows for a more rigorous development of the topics, as well as a 360◦ protractor when students manipulate rotations.

Figure 4 - NVML, Geometry, grades 9-12

4. NCTM Custom Apps

http://www.nctm.org/standards/content.aspx?id=32704

NCTM offers a series of lessons and resources in its website under the Core Math Tools link. These resources are free and do not require membership to access. Among The set of Custom Apps for geometry address 11 topics with its unique interactive tool. The first tool, *Triangle Congruence*, allows students to

> [e]xplore whether three specified side or angle measures are sufficient information to determine the existence of a unique triangle (or the congruence of two triangles). Rigid transformations are used to verify the congruence of two triangles.

Other tools in the set that offer direct application to the MCCRS include: (a) *Tilings with triangles or Quadrilaterals*, which allows students to "[e]xplore tiling the plane with congruent triangles or quadrilaterals using rotations of 180° about the midpoints of sides"; (b) *Explore* SSA, which allows students to "[e]xperiment with various side and angle measures to explore the side-side-angle condition for triangles. Determine which measurements or intervals of measurements determine no triangle, exactly one triangle, or two noncongruent triangles"; (c) *Explore Similar* Triangles, where students can "[e]xplore whether two specified side or angle measures are sufficient information to create similar triangles. A sequence of size (dilation) and rigid transformations are used to verify the similarity of two triangles. Alternatively, measures of two triangles can be displayed to check for similarity or compared directly using the mouse"; and (d) *Explore Arcs and Angles*, where students can "[e]*xplore the relationship between the measures of inscribed angles and their intercepted arcs.* Features include the dragging the points on the circle to change the arc length or angle measure and display measures of angles and arcs". It also contains a conic sections app that supports the (+) standard G.PE.A.3.

5. **Desmos** - https://www.desmos.com/calculator

Desmos is a deceptively powerful online grapher that combines the versatility of a graphing calculator with the power of visualization software. It plots equations and tables in 2D and 3D and its CAS capability allows for the use of implicit functions. Its user-friendly interface and ample documentation allows users to create and manipulate objects in no time.

6. **Illuminations** - http://illuminations.nctm.org/ActivityDetail.aspx?id=189
7. **GeoGebra** – http://www.geogebra.org/cms/en/

This is a free dynamic mathematics software. As such, it is capable of creating multiple linked representations of a plethora of mathematical objects, including geometric ones that may support the experimentation of geometric objects and facilitate constructions. This software is available in multiple versions that include options for you to: download as a free –standing program, use the online version, or download the app to Android or Apple devices. Basic capabilities resemble that of the Geometer's SketchPad™ (GSP), including access to pre-designed animations. Like the GSP, taking full advantage of this site requires investing in the construction of the interactive objects ahead of time or downloading one of the multiple user-created ones that are available on the web-site. Those who take advantage of extensive documentation or of the online community will be well served by the increased flexibility these interactive objects can provide.

8. **MAA – Exploring Geometric Transformations in a Dynamic Environment -** http://www.maa.org/publications/periodicals/loci/resources/exploring-geometric-transformations-in-a-dynamic-environment-description

Designed by Cheryll Crowe, from the Eastern Kentucky University, these explorations come with activity sheets and GeoGebra-generated applets. Focused on the study of transformations typically taught in middle and high school,

> [t]he design of the applets provides flexibility of content and grade level with hidden elements that can be revealed to create additional topics for investigating reflections, rotations, translations, and glide-reflections. Suggested activities and exercises are included but can be altered or omitted to accommodate various levels of mathematical understanding.

Each of the 8 activities in this resource helps students make conjectures based on the observations afforded by the manipulation of the applet, which only requires a browser with Java installed.

9. **PARCC Sample Items** http://practice.parcc.testnav.com/#

This site contains two areas of interest. The Sample Items portion contains sets of sample PARCC assessment items for different grade bands. When you navigate to the Sample Items portion of the site, selecting the button labeled High School Math Item Set will give you access to 10 questions purported to exemplify student constructed response items aligned with the CCSS-M for high school mathematics. Figure 1 below depicts a geometry question that integrates the animation of a construction that prompts students to use their deductive reasoning skills to write the proof that supports the construction. This is a clear example of how the PARCC exam intends to integrate media into the assessment. It also demonstrates how fluency with construction will be leveraged to help students demonstrate proficiency with the Standards for Mathematical Practice.

Figure 5 - Sample Geometry Assessment Item

In the Practice Tests portion, you are able to select the mathematics practice test from the drop down menu, also available at http://practice.parcc.testnav.com/#, to access a sample test. The Geometry End of Year test contains another 32 questions, 7 of which are non-calculator. This may also be a worthwhile experience for your students.

Conclusion

The Common Core holds the potential to realize "[t]he twin goals of equity and high-quality schooling have a profound and practical meaning for our economy and society" (Freire, 1970, p. 13). However, while the high-quality materials are not fully aligned to the MCCRS, teachers are not at the mercy of book publishers. As demonstrated in this article, there is a wealth of online tools that can be infused in the enterprising teacher's classroom to support student academic development and achievement.

Moreover, to ensure the quality of the resources being developed are consistent with the expectations of the MCCRS, teachers should also review the NGACCSSO's guidelines for the production of teaching resources. These guidelines assist teachers in the creation of resources "ranging in length from an individual problem set or lesson up to an entire unit or longer. States, districts, schools, and teachers themselves can use the criteria to assess the alignment of teacher-developed materials to the Standards and guide the development of new materials aligned to the Standards" (NGACSSO, 2012, p.6). Additionally, the high school guidelines are organized around the shifts, the standards, and the achievements in grades K-8. Both sets of criteria acknowledge the fact that instructional materials may exist in digital form and that those digital and online materials and tools may leverage their unique navigational features to promote focus and coherence when designed properly (NGACSSO, 2013, p.1).

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