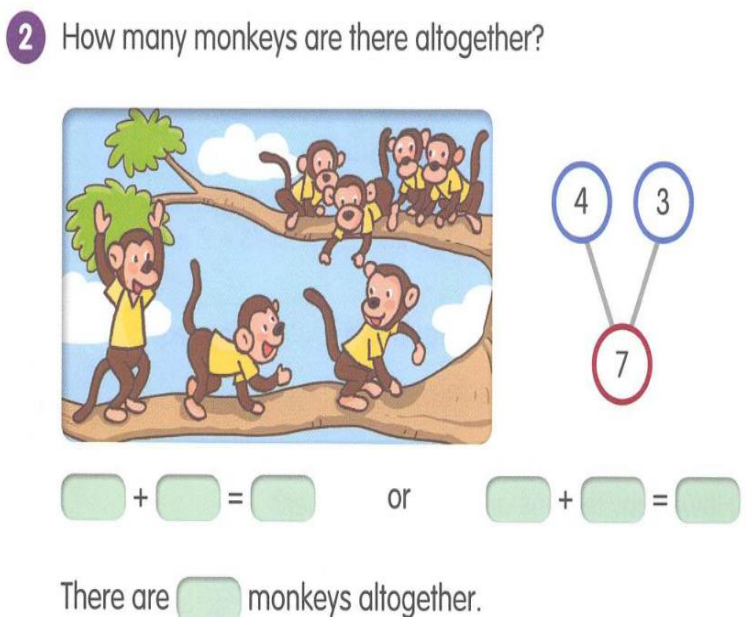
Fostering Algebraic Thinking for First Graders: Insights from Shanghai and Singapore*



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Stevenson University

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Fostering Algebraic Thinking for First Graders: Insights from Shanghai and Singapore

The Structure

I. Introduction
II. The two textbooks
III. The Six Measures of Fostering Algebraic Thinking for First Graders
IV. Conclusion
V. Discussion

from	to
3 – 1 = 2 + 1 =	3 - 1 = x (working with symbols) y = 2 + 1 (new way of presentation) 1 + n = 3 – n (shifting to a structural operational perspective) ……
Alex has 2 candies, and Bob has three times as many candies as Alex. How many candies do they have in total? (2+3x2=8)	Alex has n candies, and Bob has m times as many candies as Alex. How many candies do they have in total? (use logic reasoning to generalize) (n+mn)

Algebraic thinking from **elementary school** as the solution to solve this cognitive and psychological challenge.

No consensus on how early to start it nor the specific content of algebraic thinking. And the outcomes of the implementation have varied greatly.

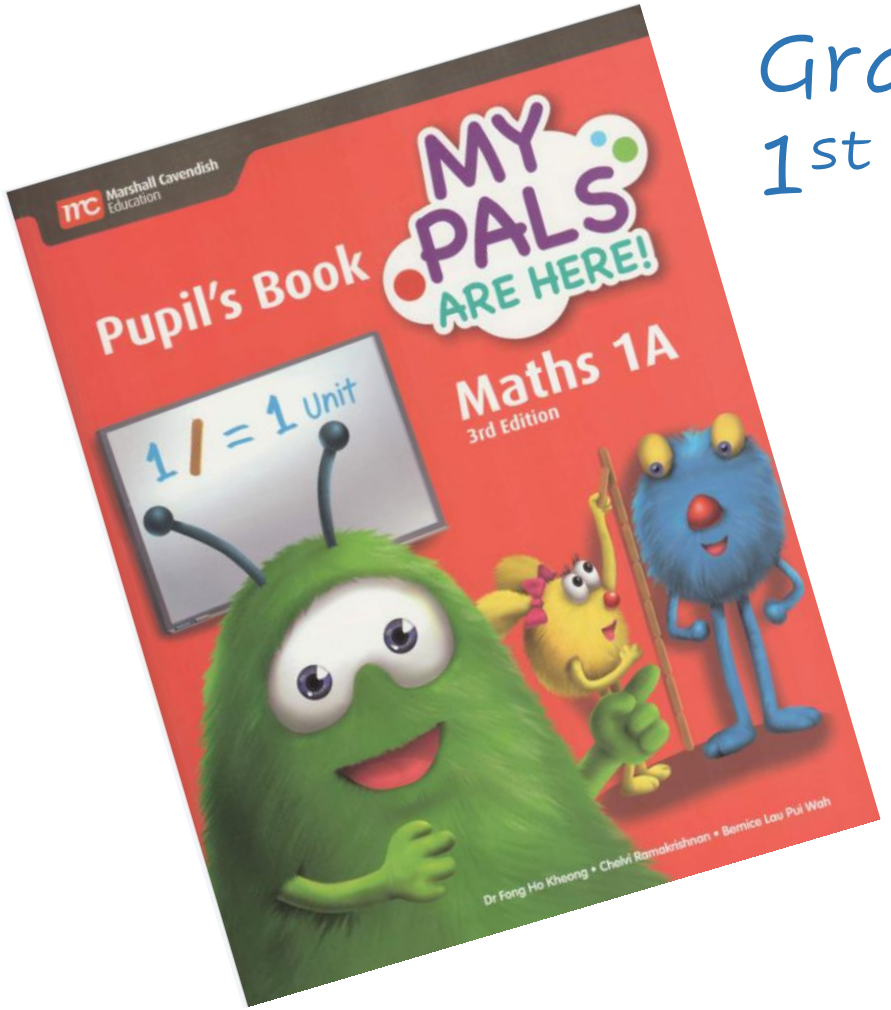
questions remain ...

- How early?
- What's the content?
- How do teachers balance the workload?

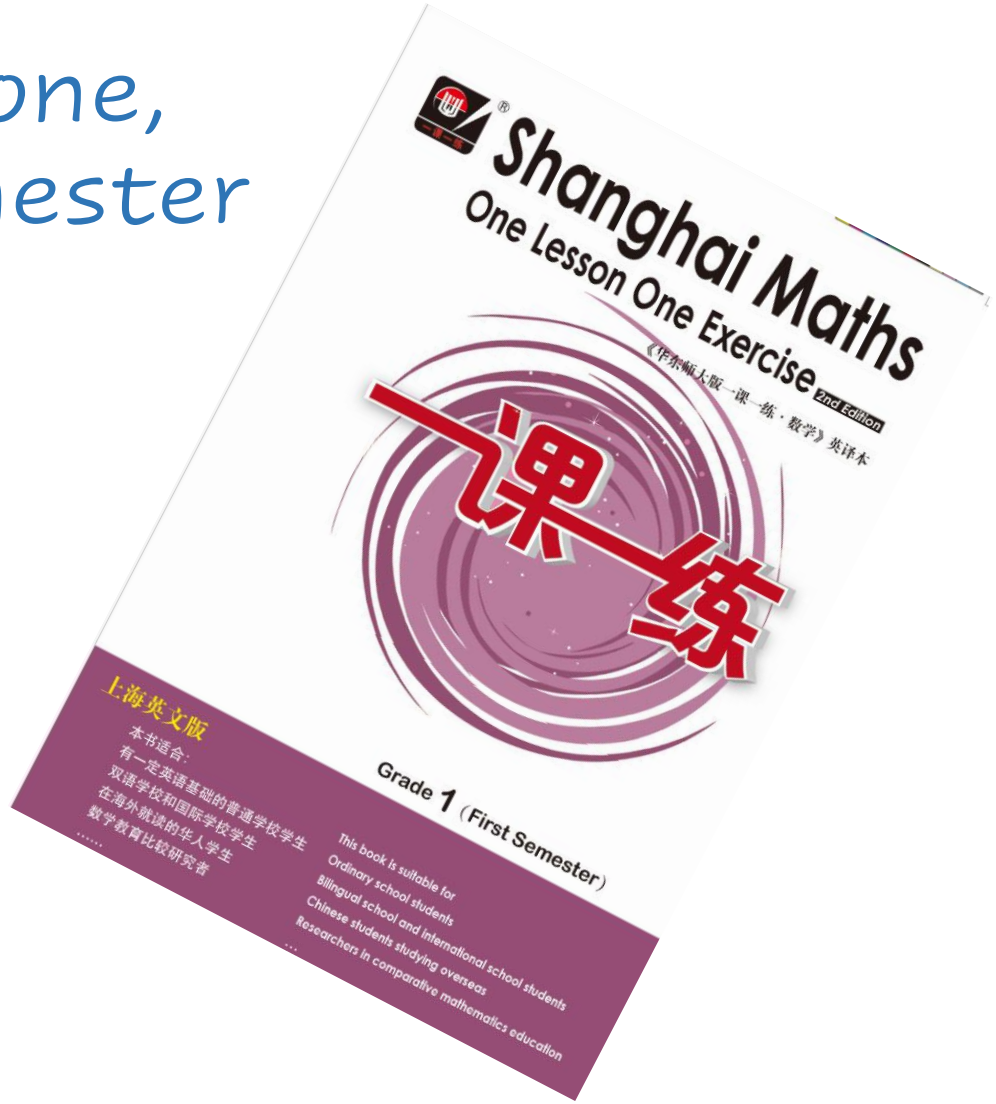
Today, we are going to look at how Singapore & Shanghai synchronize arithmetic teaching and algebraic thinking in the first semester of Grade 1.

- The results of the international assessments like TIMSS and PISA, show success stories from the Eastern Asian areas.
- A content-specific analysis reveals that these teams are very strong in number and algebra (Mullis et al., 2019).
- Singapore and China are two such cases

II. The two textbooks



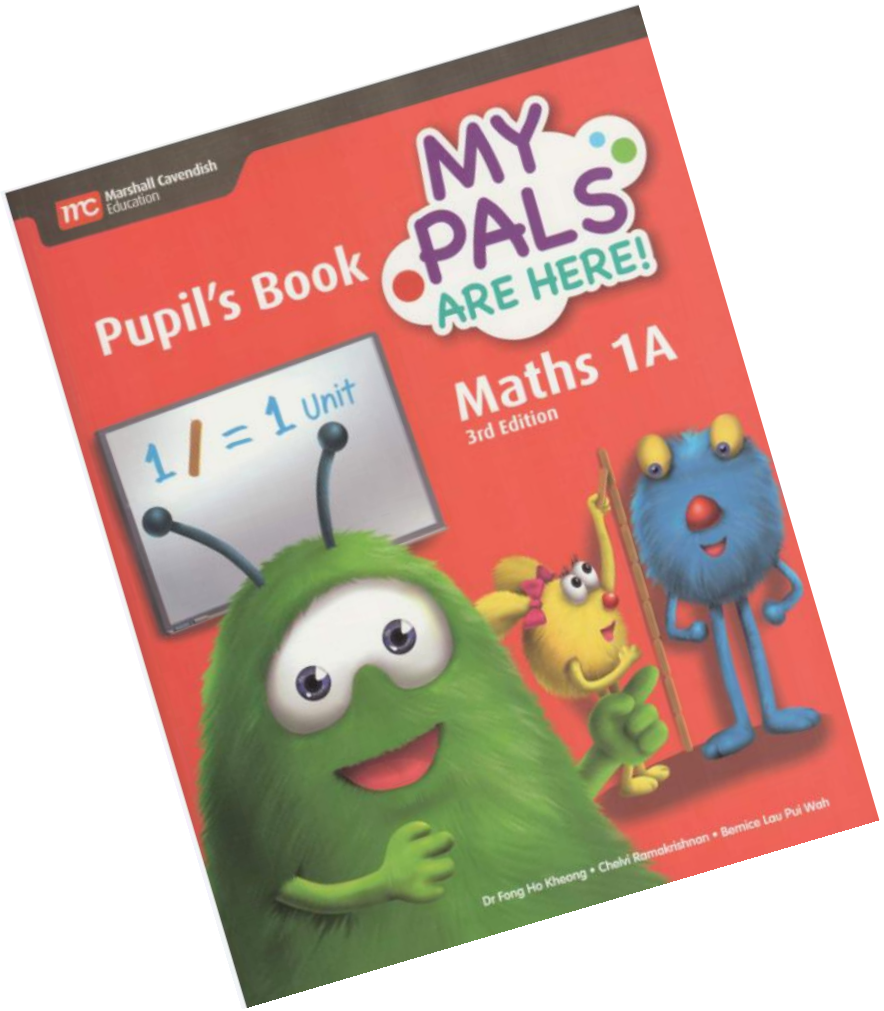
Grade one,
1st semester



Kheong, F.H., C. Ramakrishnan, & B.L.P. Wah (2015). Marshall Cavendish Education

Fan L. H. et al. (2020) East China Normal University. East China Normal University Press.

II. The two textbooks

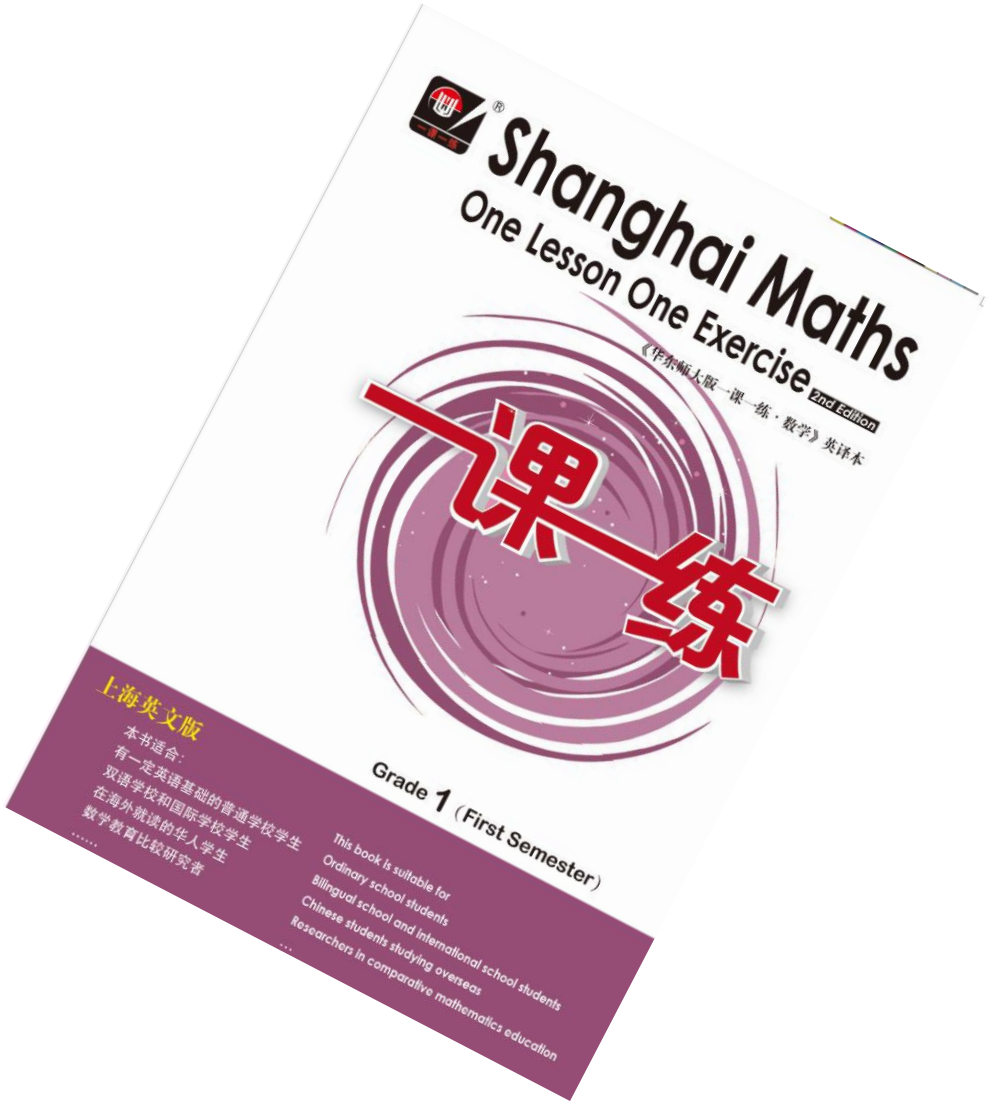


- align with the Singapore Ministry of Education's syllabus
- widely adopted and highly popular amongst teachers and pupils in Singapore

Kheong, F.H., C. Ramakrishnan, & B.L.P. Wah (2015). Marshall Cavendish Education

II. The two textbooks

- Shanghai Maths' is aligned to China's National Curriculum.
- Shanghai Maths' has been used and updated in China over 30 years, Its materials have been tried and tested by teachers and students (Collins)
- A revised version has been adopted by the U.K. since 2013. Five years after that, UK's Math PISA results has been rising from below the OECD average to above it (OECD 2023).



Fan L. H. et al. (2020) East China Normal University. East China Normal University Press.

III. The Six Measures of Fostering Algebraic Thinking for First Graders

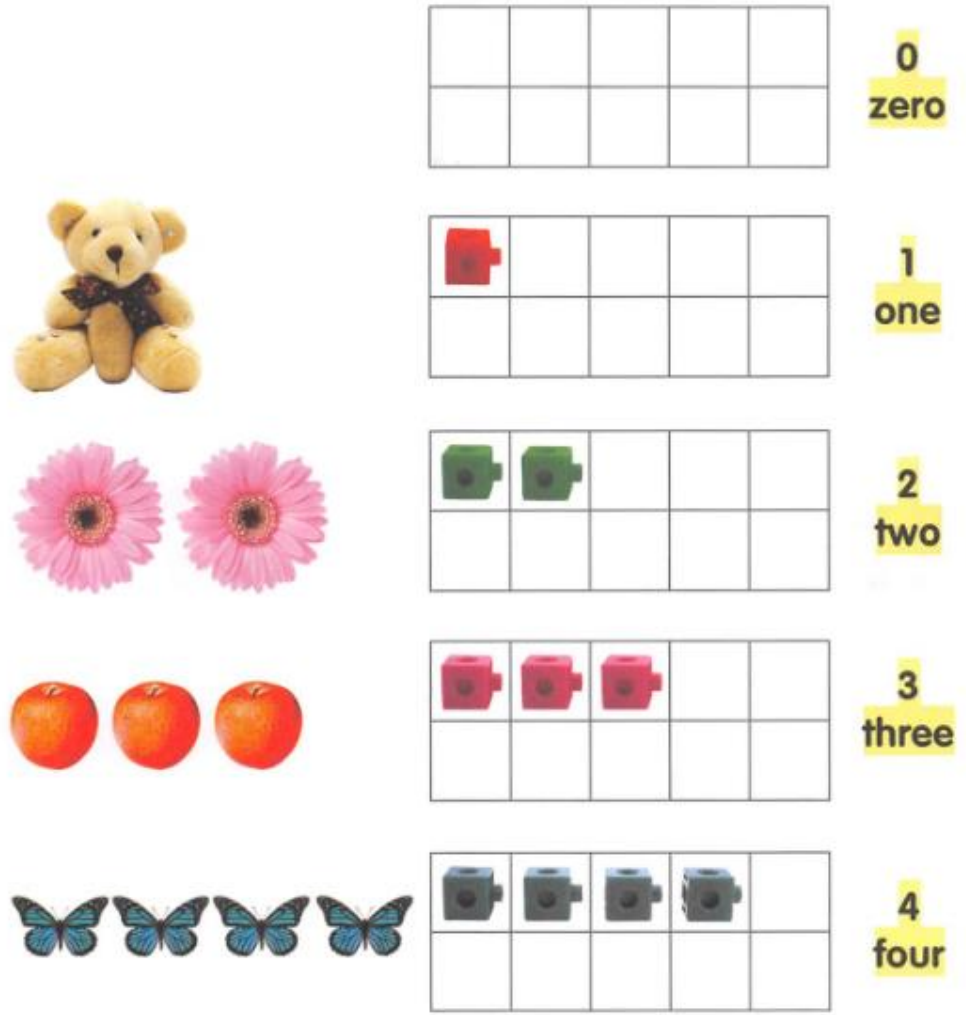
1. One-sheet strategy
2. Learning equal and unequal signs simultaneously
3. Variation
4. Making writing number sentences a habit
5. Befriending symbols
6. Use of good analogies

Visualize connections by putting related items as a single question on a single sheet

Why numbers are abstract?

Because 3 can be anything with a quantity of 3, no matter three apples or three cats, as long as there are three of them.

Fig. 1a: One - sheet strategy to show CPA



Visualize connections by putting related items as a single question on a single sheet

pupils connecting

CPA themselves

Fig. 1b: One - sheet strategy to do CPA

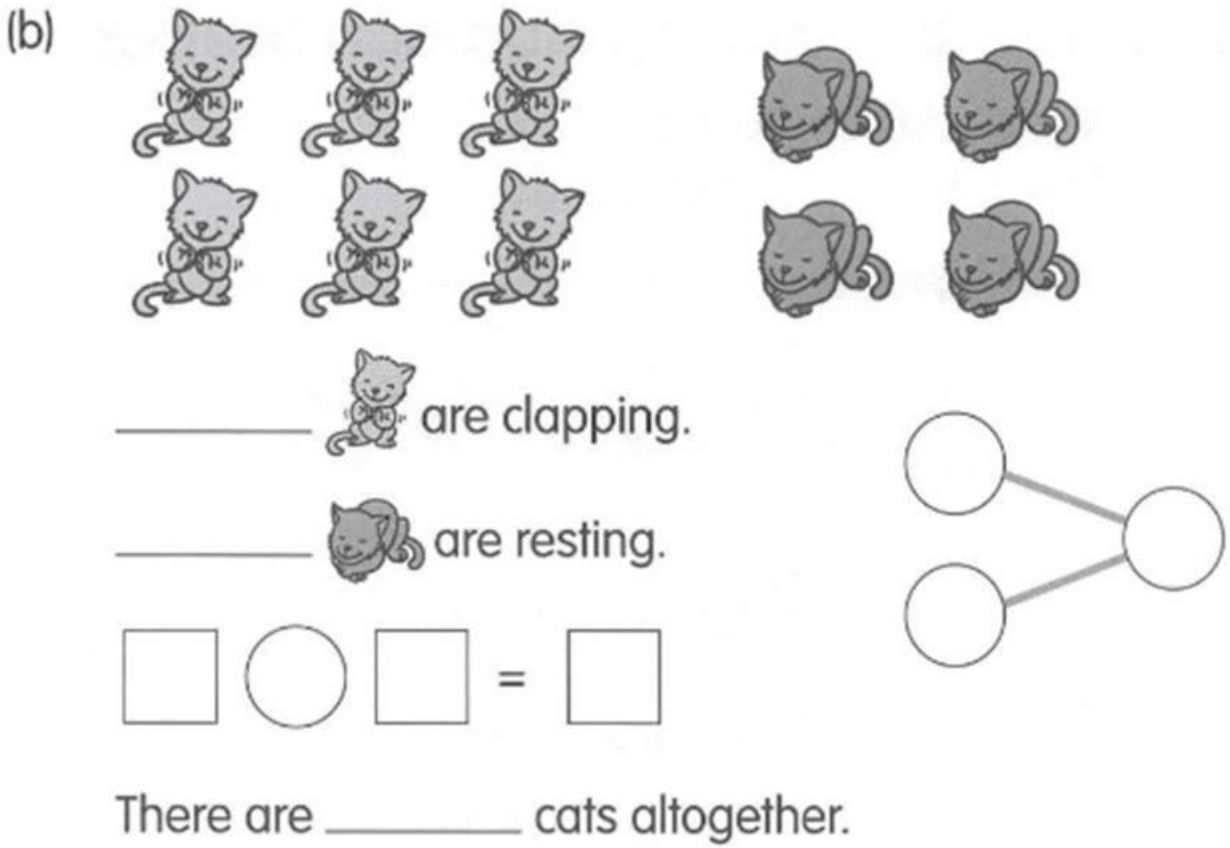
3 Count, colour and fill in the spaces.

3					

Visualize connections by putting related items as a single question on a single sheet

Showing the understanding of the problem visually and formally

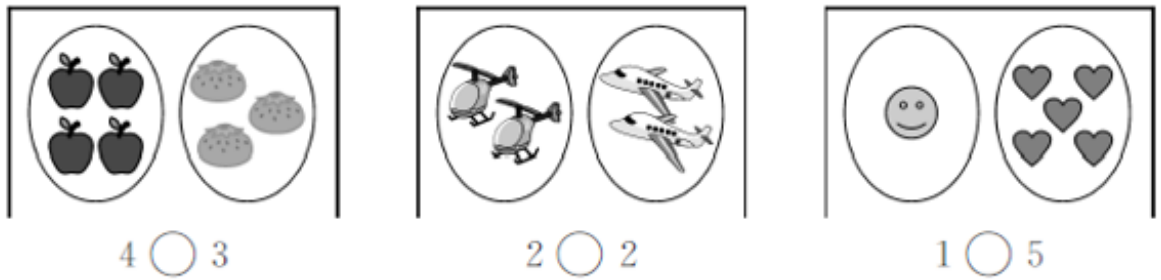
Fig. 1c: One- sheet strategy for addition



By learning of '=', '<', and '>' at the same time, the true meaning of the equal sign is demonstrated

Fig. 2a: Learning Equal Sign & Inequal Signs simultaneously (I)

Q1. Look at the pictures, and fill in each ◯ with '>', '<', or '='.



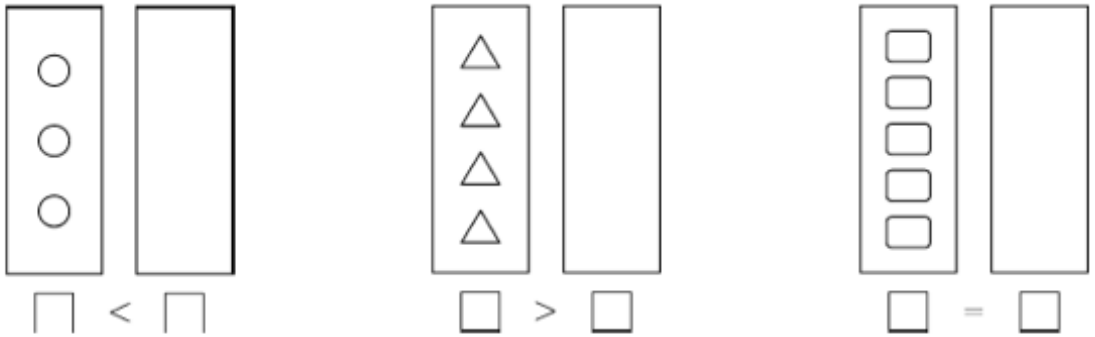
It is the quantity that matters …

Q 2. Look at the pictures and fill in the spaces.

By learning of '=', '<', and '>' at the same time, the true meaning of the equal sign is demonstrated

Fig. 2a: Learning Equal Sign & Inequal Signs simultaneously (I)

Q 3. Compare, draw and then fill in the boxes below.



Finaly, its all about the relationship b/t the quantities of both sides.

Q4. Fill in the □ with numbers and the ◯ with '>', '<', or '='

6 ◯ 7	□<4	5=□	7 ◯ 3 ◯ 1
□>4	2 ◯ 2	9 ◯ 3	4<□<6
□=□	□<□	□>□	6>2 ◯ □

By varying problems sets, the master of a mathematical concept or a procedure is deepened

Before starting the topic of subtraction, pupils see a 'message' from subtraction

Fig. 3a: Variation to understand inverse relationship

Q 3. Let's calculate

$2+3=$	$5+4=$	$1+4=$	$6+\square=8$
$4+0=$	$3+7=$	$4+2=$	$\square+4=9$
$3+3=$	$5+2=$	$3+6=$	$\square+7=7$
$6+4=$	$3+4=$	$4+5=$	$8+\square=10$

By varying problems sets, the master of a mathematical concept or a procedure is deepened

Extending to a structural operational perspective …

Fig. 3b: variation to progress to structural perspective of operation

Q4 Fill in the brackets

$(\quad)+4=8$ $2+(\quad)=10$ $7-(\quad)=3$

$3+(\quad)=6$ $(\quad)+8=8$ $(\quad)-6=3$

$5+(\quad)=9$ $(\quad)-3=6$ $8-(\quad)=10-4$

By varying problems sets, the master of a mathematical concept or a procedure is deepened

Consolidating the concept of '=' and mind-setting for equations…

Fig. Fig. 3c: Variation to enhance structural understanding of the equal sign

Q6. Fill in the ◯ with '>', '<', or '='

$7+3 \bigcirc 8$	$5+4 \bigcirc 7$	$2+7 \bigcirc 5+3$
$4-2 \bigcirc 6$	$9-3 \bigcirc 6$	$5+3 \bigcirc 9+0$
$2+8 \bigcirc 10$	$8-5 \bigcirc 10$	$6+4 \bigcirc 5+5$

Number sentences formalize the expressions of understanding and are a foundation for algebraic expressions..

Before giving the answer, translate the problem mathematically…

Fig. 4a: Writing number sentence to translate word problem into formal mathematical expression

(2) 6 children are on the merry-go-round.
6 more children join them.
How many children are there now?

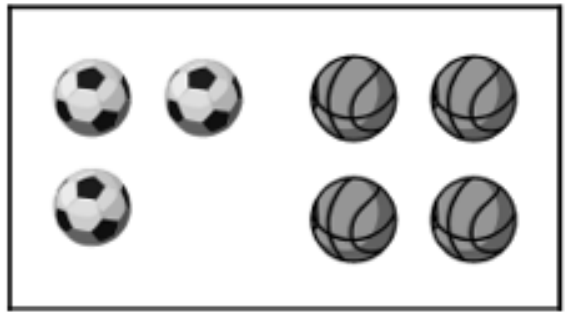
□ ○ □ = □

There are _____ children now.

Number sentences formalize the expressions of understanding and are a foundation for algebraic expressions.

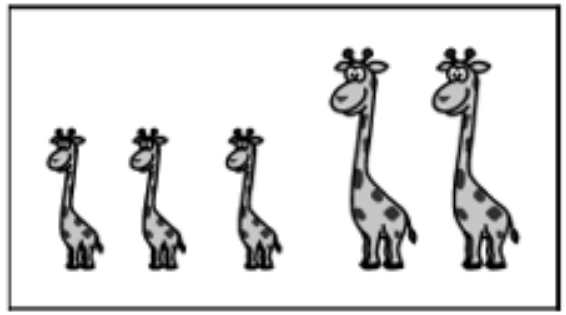
Fig. 4b: Writing number sentence to discover commutative property for addition

Q2 Look at the pictures and fill in the boxes (section)



□ + □ = □

□ + □ = □



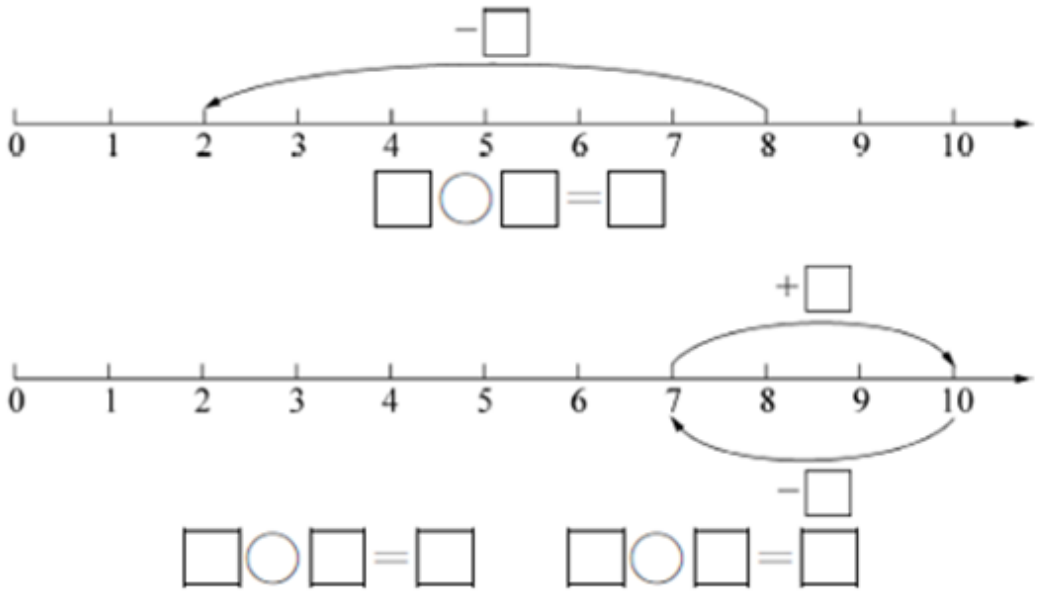
□ + □ = □

□ + □ = □

Why one situation can be translated into two number sentences?

Number sentences formalize the expressions of understanding and are a foundation for algebraic expressions.

Fig. 4c: Writing number sentences to express the understanding of a number line



Number line is another way to understand addition and subtraction

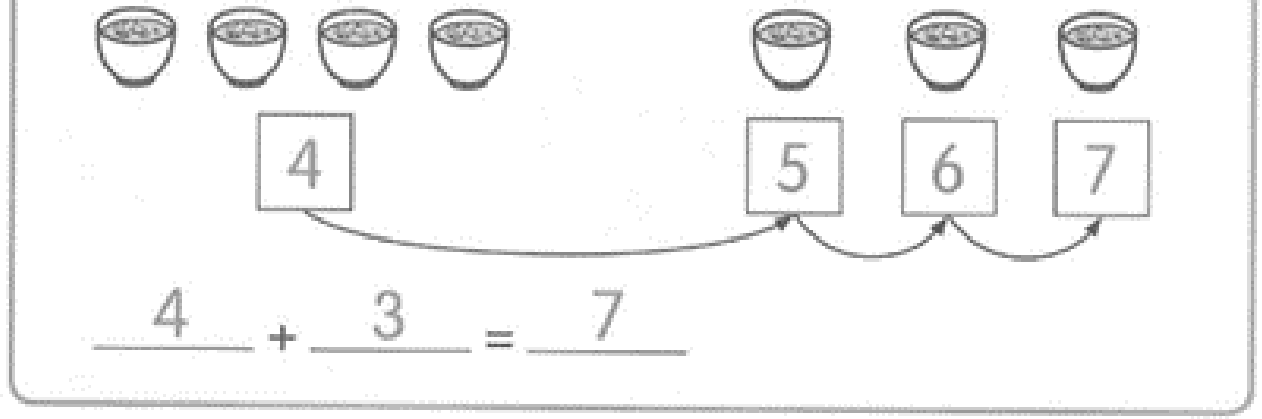
Use symbols that children are comfortable with and, when they are replaced by *x* and *y*, they will feel less strange.

Each box is a missing number

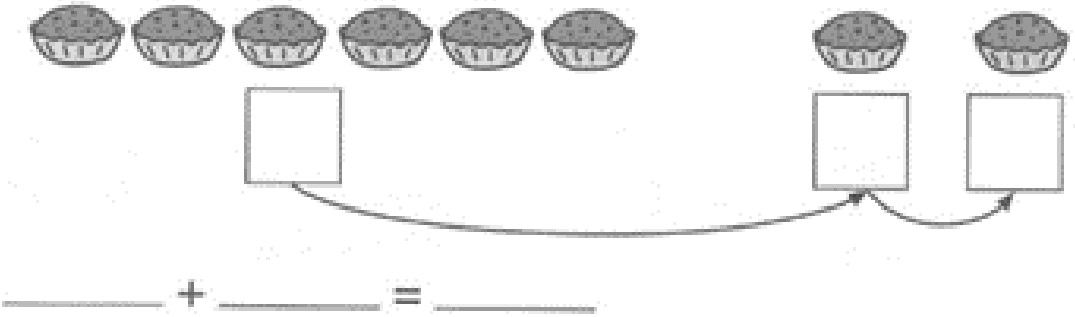
Fig. 5a: Boxes as symbols

(2) Add.
Count on from the greater number.

Example

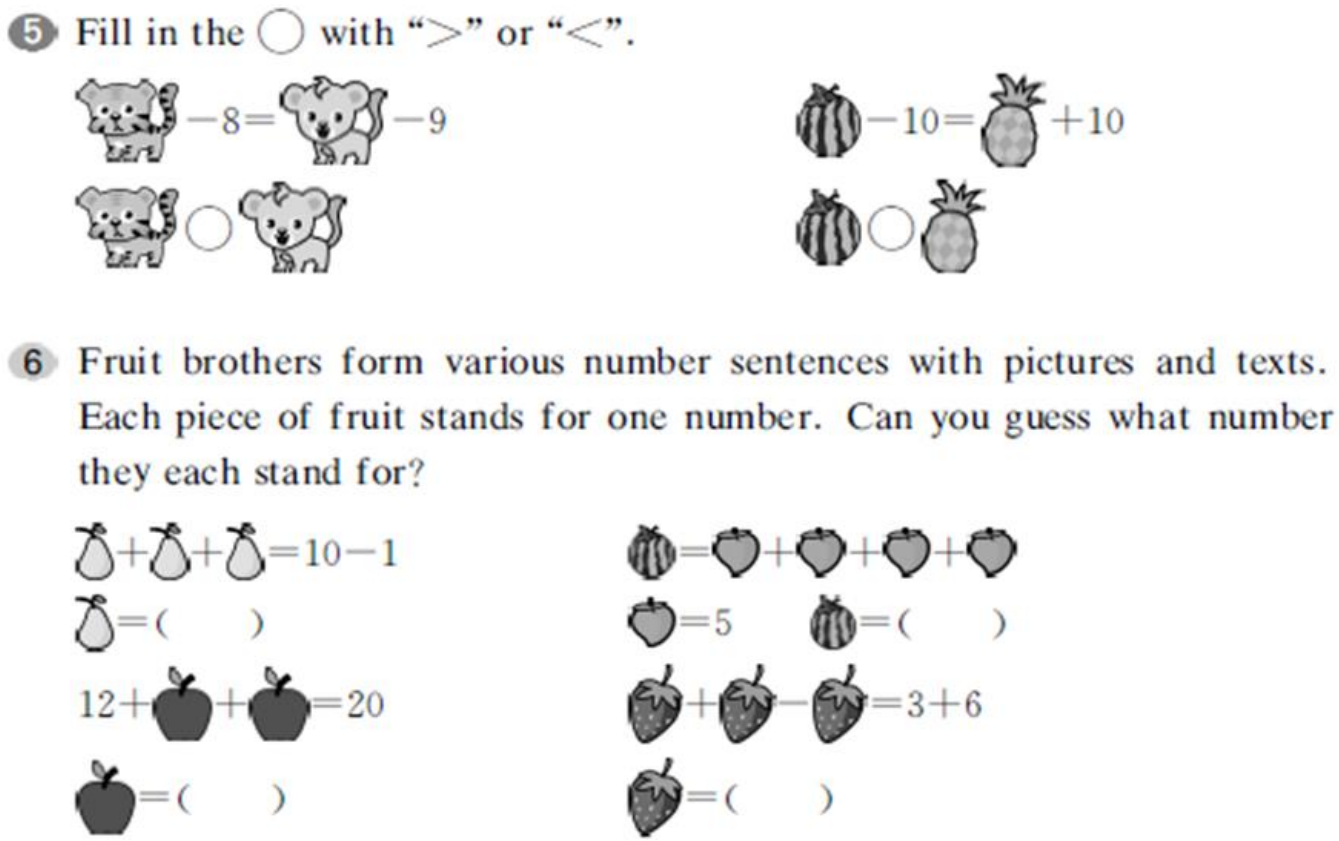


(a)



Use symbols that children are comfortable with and, when they are replaced by x and y, they will feel less strange.

Fig. 5b: icons as symbols

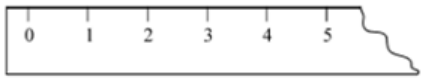


Each icon represents a number

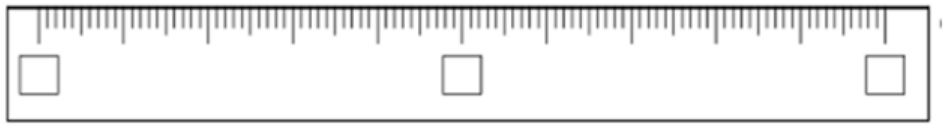
Good analogies lay a good foundation for future learning of algebra.

Fig. 6: Ruler as a good analogy for learning number line

Q4. Look at your ruler and find where '0' is located. Can you think what it stands for?



Q5. Fill in each ☐ with a suitable number



The number line inherits the main characteristics of a ruler: an origin, direction, and equal spacing

Q1 Which of the following can show a complete number ray? Put a V in the ( )

(1) 0 1 2 3 4 5 6 7 8 9 10 ( )

(2) 0 1 2 3 4 5 6 7 8 ( )

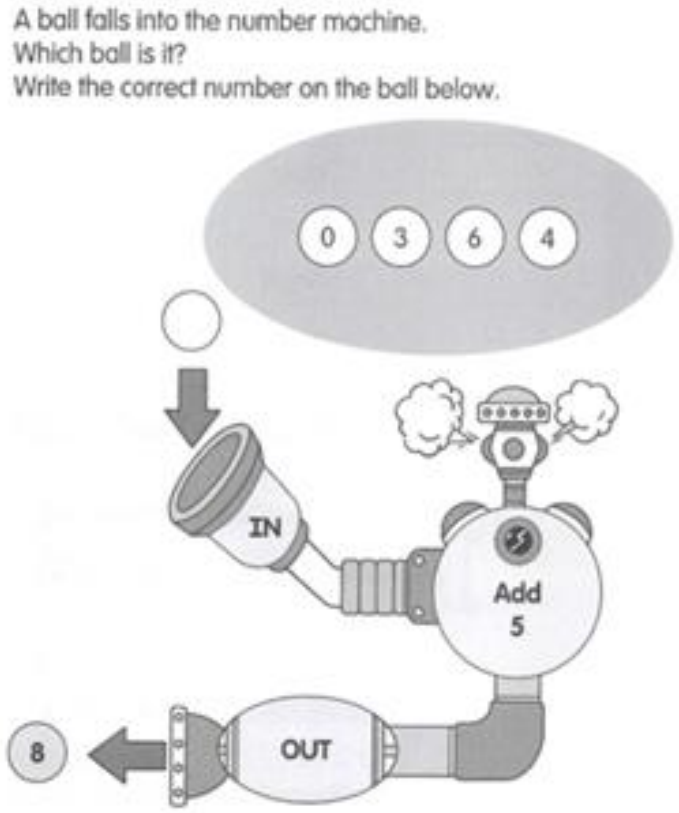
(3) 1 2 3 4 5 6 7 8 9 10 11 ( )

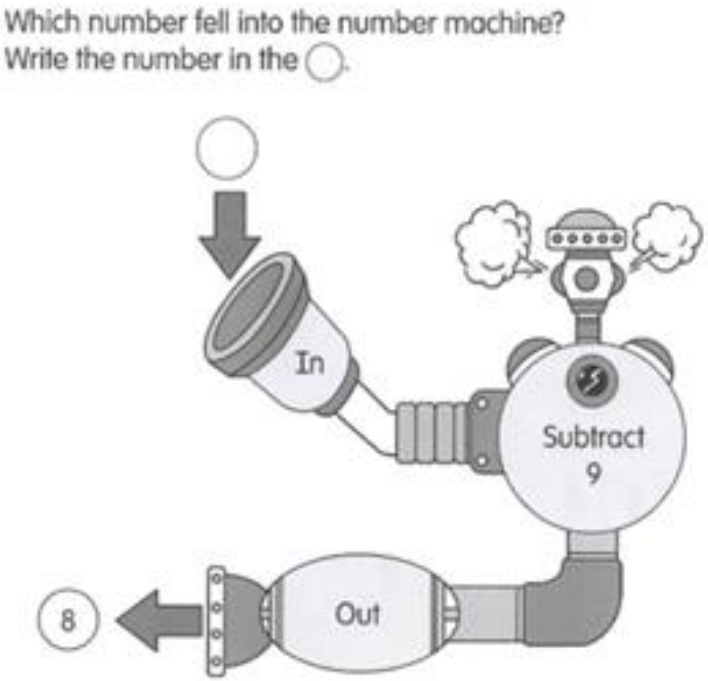
(4) 0 2 4 6 8 10 ( )

Good analogies lay a good foundation for future learning of algebra.

Fig. 6.2 Number machine as a good analogy for understanding one-to-one relationship

The number machine is often used in middle schools to practice substitution and introduce functions





IV. Conclusion

1. For First graders, it is not about directly teaching algebra, but rather, laying the groundwork for it

- establishing connections between the concrete and abstract.
- building understanding of important concepts such as inverse operation, commutative property for addition, equal sign and number line (ray).
- preparing students' algebra learning by familiarizing them with structural perspective, symbols and number machine.

IV. Conclusion

2. Pedagogically, it is not about adding extra teaching load, but rather, using effective methods to achieve the synchronous teaching

- one -sheet strategy
- variation
- simultaneously teaching ‘=’, ‘<’, and ‘>’
- insisting formalizing answers by writing number sentences
- using a ruler to approach number line

IV. Conclusion

3. In the First semester of Grade one, don't rush pupils to learn more numbers but rather, fully utilize the simple numbers within 20.

The focus on a smaller range of numbers enables pupils to develop a deeper understanding of fundamental mathematics with **minimal computational burden**. It proves that this approach helps pupils learn content faster and better in later grades.

IV. Discussion

" Chinese mathematics education adopts an inclusive approach, constantly integrating various excellent educational concepts from around the world through both theoretical analysis and practical testing” (Zhang and Liu, 2020).

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Thank you