Lesson Seed

Exploring Fractions Concepts with Virtual Fraction Bars

Drew Polly, University of North Carolina Charlotte

Background: The Benefit of Virtual Manipulatives

Learning technologies hold mathematical power to enhance the teaching and learning of concepts in multiple ways. Specifically, learning technologies that are open-ended learning environments (OELEs) provide students with a workspace and dynamic mathematical tools that they are able to use to examine mathematical situations and generate representations of those situations. One type of OELE is virtual manipulatives. Moyer-Packenham and Bolyard (2016) define virtual manipulatives as “an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge” (p.13). As is the case for all mathematics virtual manipulatives, teachers can use the tool in various ways, ranging from unstructured exploration of the tool to using the tool to solve a specific task. This lesson seed provides examples of both unstructured use of the tool as well as a specific activity that uses the tool to support teaching and learning.

Research on the use of technology, including virtual manipulatives, in mathematics classrooms shows that technology can support teaching and learning when it is used in ways linked to mathematical reasoning, problem solving, and higher-level thinking skills (Howerton & Polly, 2023; McCulloch & Lovett, 2023; Zbiek et al., 2007). Additionally, studies have found that the use of virtual manipulatives can greatly benefit student learning (Moyer-Packenham & Westenskow, 2013; Polly, 2016; Polly & Martin, 2024a; Urbina & Polly, 2017).

Lesson Seeds about Virtual Fraction Bars in Fourth Grade

The next section includes lesson seeds (vignettes) that describe the use of Virtual Fraction Bars on Polypad (https://polypad.amplify.com/p/HytFJ80HcN7zQ#fraction-bars), virtual mathematics manipulatives that allow for the exploration of fraction concepts. These lesson seeds and vignettes originate from two different fourth-grade classrooms with students who vary in terms of both socioeconomic status and ethnicity. Data from a recent assessment on fractions indicated that approximately half of the class was still working on foundational fraction concepts from previous grade levels while also engaging in learning experiences with grade-level concepts. The teacher had taught third, fourth, and fifth grade for 22 years and through our interactions had demonstrated a sound understanding of all of the fraction concepts in those grades. The author, who works regularly with the teacher, was there to support the activity.

Vignette 1: Using Virtual Fraction Bars to Explore Decomposing a Fraction

Launch of the Task

One way to use virtual manipulatives in classrooms is to allow students to use it as a tool to explore open-ended complex tasks that require reasoning and may have multiple correct answers. This lesson took place on the third day of the fraction unit. During the first two days students had used the virtual fraction bars to explore equivalent fractions with halves, fourths, and eighths, which is an extension from concepts they learned in third grade. This lesson started with the fourth-grade teacher displaying the following task:

Exploring Fraction Concepts *(continued)*

Part 1: I have a box of 12 cookies. How many different ways can I repackage the cookies into two different bags? What fraction of the box is in each of the bags?
Part 2: When the cookies are repackaged, determine which bag has more than half or less than half.

The teacher launched this task by asking students questions from the Three Reads Protocol (San Francisco Unified School District, n.d.). During the first read, the teacher read the task and asked students, "What is this task about? Go ahead and talk with your partner." Students then briefly said things such as, "This task is about cookies," and "We have to take 1 bag of cookies and put it into 2 different groups."

After a minute during the second read, the teacher reread the task and asked, "What does each number mean in the problem?" The teacher then had students share and facilitated a class discussion where students shared the following statements:

- *There are 12 cookies in the box.*
- *We need to split the cookies into two bags.*
- *We need to see which bag has more than half of the cookies. I know that half of the box is 6 cookies since 6 plus 6 is 12.*

While the protocol gives teachers the flexibility to read the task a third time and then ask more questions the teacher decided to let students begin exploring. She said, "Using the virtual fraction bars I want you to go ahead and begin working."

Student Exploration of the Task

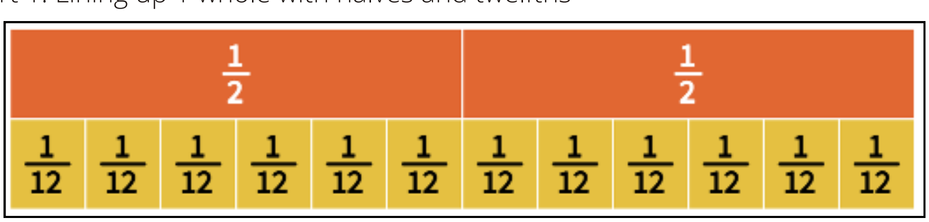
Students worked in pairs, sharing a device, and all of the students moved the fraction bar with 12 twelfths into the workspace. When asked why they chose twelfths, a pair of students said, "The box has 12 cookies so 1 cookie is 1/12 of the box." Another pair of students commented, "We have 12 cookies, so it made sense to use the 1/12 pieces." Once students moved the fraction bar with twelfths on it the teacher asked, "Is there anything else in the problem that we may want to include in our representation?" Students immediately made comments such as, "We should use the bar with halves on it," but their explanation only referred to "The problem says 1/2 so I think we should use the half fraction bars."

Once students had both the 1/12 and 1/2 bars, students then used the split button located on the fraction bars to decompose them into smaller pieces. They then moved the pieces and explored finding combinations that worked. Figure 1 shows an example. Part 1 of Figure 1 shows the fraction bars lined up. Part 2 of Figure 1 shows that the pair of students decomposed 1 whole into 5/12 and 7/12. By using the halves and lining up the fraction bars they could tell whether 5/12 and 7/12 were less than, equal to, or greater than one-half.

Figure 1

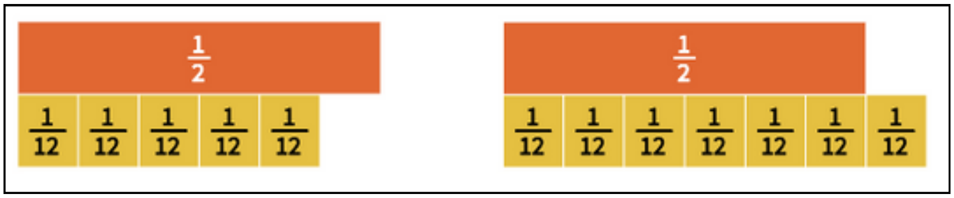
Screen shot from a pair of students

- Part 1: Lining up 1 whole with halves and twelfths



Exploring Fraction Concepts *(continued)*

- Part 2: Decomposing the 12 twelfths into 5/12 and 7/12



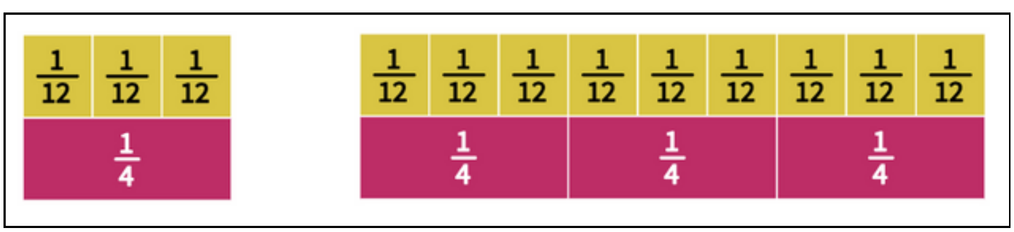
Students continued to explore different ways to break up one whole into twelfths. This was the first day that students had explored twelfths. The teacher circulated and talked to students. After students the teacher gave them an extension task:

Take a look at your solutions and see how each of them compares to fourths.
Just write down some ideas about what you notice.

Here is a conversation that took place between the teacher and two students during the extension activity.
Teacher: Tell me what you did.

- *Student 1: Like you asked we looked at our solutions and thought about how twelfths compare to fourths.*
- *Student 2: In this one (points to the picture in Figure 2), we found that we can make fourths out of twelfths.*
- *Teacher: You said you can make fourths? What do you mean?*
- *Student 2: Look at this. When we line up three twelfths, that is the same size as one-fourth, so they are equal. That means that if I need to make a fourth, I could do that by putting three twelfths together.*
- *Teacher: Do you notice anything about the other twelfths in your picture?*
- *Student 1: I see that nine-twelfths is the same size as three-fourths.*
- *Student 2: So, nine-twelfths is equal to three-fourths.*

Figure 2
Screenshot relating twelfths to fourths



All of the pairs of students were able to correctly decompose twelfths into different pairs of fractions and successfully compare them to one-half. While the teacher intended for this exploration to focus on decomposing fractions (4.NF.B.3 in Maryland and Common Core State Standards), this lesson had natural extensions to equivalent fraction concepts (4.NF.A.1 in Maryland and Common Core State Standards).

Exploring Fraction Concepts *(continued)*

Vignette 2: Using Virtual Fraction Bars to Answer a Fraction Comparison Task

Launch of the Task

The lesson began by having students debate the following task:

I need at least 2/3 of a cup of flour to make pancakes.
My flour container holds 1 cup and there is 1/4 of a cup of empty space.
Do I have enough flour?

The teacher displayed this task, read it out loud to the class, and then said, "Please have a conversation with your table about how you would use math tools or drawings to explore this task." The teacher circulated and listened to conversations. Some of the comments from students included:

- *We need to draw pictures on graph paper.*
- *We can just look at the denominators to figure out which piece is larger.*
- *We can find equivalent fractions and once the denominators are equal, we can compare the numerators.*

The teacher invited a few students to share their ideas with the class. Sensing that some students had some uncertainty about the idea of empty space, she asked, "What does it mean that there is 1/4 of a cup of empty space?" Students hesitated at first and did not answer. The teacher then said, "If there is 1/4 of a cup of empty space, do we know how much flour we have?" She continued, "As you start to explore this task, that may be something to think about."

The teacher then gave students access to their Chromebooks so that each pair had access to one device. Students were instructed to open up the Virtual Fraction Bars app. The teacher said: "While we talked about various ways to explore this task, I would like for you to solve the task two different ways. First, I want you to use the virtual fraction bars. After that you can use a different strategy on paper to check your work."

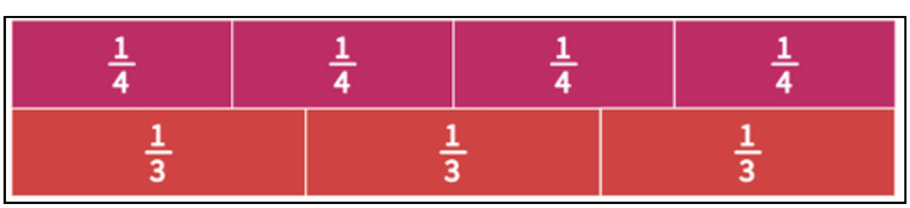
Student Exploration of the Task

As students worked in pairs on this task they used the virtual manipulatives in various ways. One of the two primary ways included keeping the fraction bars connected and lining them up (Figure 3). When they were lined up and the teacher asked students what they were looking at here are some responses:

- *I know that I need two-thirds. I need to see how that compares to fourths, so I kept the fraction bars connected.*
- *I know that since four-fourths equals a whole and I have one-fourth of empty space I have three-fourths of a cup.*
- *I know that if I disconnect the fraction bars, they may look unequal, but they need to look equal since four-fourths and three-thirds both equals a whole.*

Figure 3

Screenshot from students who kept fraction bars connected



Exploring Fraction Concepts *(continued)*

Another strategy that students used was to split the fraction bars into individual pieces and then line them up (Figure 4). In this case, students were able to orally tell the teacher the amount of flour that they had. One student whose screen looked like Figure 4 commented, "I know that four-fourths is a whole. If there is one-fourth of empty space, then I can take one-fourth away. That means I only have 3 one-fourth pieces which is three-fourths."

Figure 4

Screenshot from students who split fraction bars

Students who did this talked to the teacher about the benefit of being able to keep track of the value of the fraction bars more easily. One pair of students who sometimes demonstrates a need for additional learning opportunities had the following conversation with their teacher:

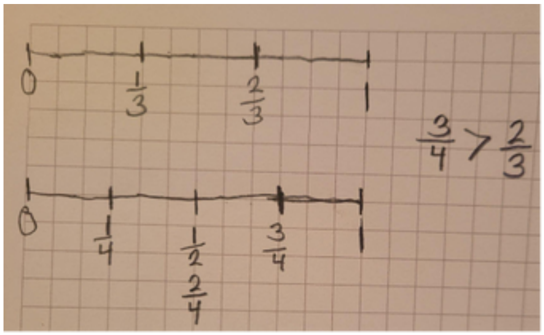
- *Student 1: We know that we needed two-thirds, so we moved two pieces of one-third. We then moved all of the fourths out but took one of the fourths away since that was the amount of extra space.*
- *Teacher: How many fourths did you have after you took one-fourth away?*
- *Student 2: Let me count again. Student counts the three one-fourth pieces.*
- *Student 2: We have three-fourths. It looks like three-fourths is more than two-thirds.*
- *Teacher: So, if we go back to the problem how does this picture help us?*
- *Students look at each other and do not respond.*
- *Teacher: Why don't you look at your picture and see what the question is asking?*
- *Student 1: We know we need two-thirds of a cup, and it looks like we have three-fourths. Based on that picture three-fourths is more than two-thirds so we should have enough.*
- *Teacher: [Student 2] can you revoice that in your own words?*
- *Student 2: Three-fourths is more than two-thirds in our picture, so we have enough.*
- *Teacher: Go ahead and prove your answer using a different strategy.*

As students worked on their second strategy most pairs of students made the exact same representation on graph paper as the fraction bars on the virtual manipulative. There were three pairs of students who made two number lines that were each 12 squares wide on graph paper. They plotted both 2/3 and 3/4 to determine that was larger 3/4 and that they had enough flour.

Exploring Fraction Concepts *(continued)*

Figure 5

Number line comparing 2/3 and 3/4



The following is an excerpt of the conversation that ensued when the teacher asked one of the pairs to share their work:

- *Teacher: Tell me what you did.*
- *Student 3: We ended up multiplying the denominators 3 and 4 together to get 12 so our number line is 12 squares long. We then figured out where half was which let us mark fourths since a fourth is half of a half.*
- *Student 4: I think thirds was hard. We knew we had to break the twelve squares up into three equal parts, so we ended up trying three squares which didn't work and then we tried four squares which worked. Now we know that makes sense since twelve divided by three is four.*
- *Teacher: So, based on the fact that you have both fractions on the number line how did you answer the question?*
- *Student 3: The largest fraction is the one farthest from zero, so we know that three-fourths is larger, so we have enough flour.*

Students continued to work with the virtual manipulatives and their second strategy until nearly every pair of students had explored the problem two different ways. All pairs of students successfully determined that they had enough flour since 3/4 is larger than 2/3.

Carrying this Work Forward

Both vignettes provided examples of how the virtual fraction bars provided learning opportunities to this class of fourth-grade students. The virtual fraction bars are an alternative to hands-on concrete fraction bars and provide a few benefits. First, students were able to quickly represent one whole using different denominators and use the fraction bars efficiently by using the split button and moving the pieces in the workspace. Whereas, oftentimes in classrooms, students spend time looking for all the different fraction bar pieces at the beginning of a task which leads to less time to actually explore and learn math concepts. Second, in Vignette 2, all pairs of students (even those that typically demonstrate challenges with fractions and other mathematics concepts) were able to correctly use the fraction bars with relative ease to make a mathematical representation and line the pieces up to compare fractions.

Students found the virtual fraction bars to be easy to use, and the teacher commented about how they are a quick go-to. Weeks after the vignettes featured in this article occurred, students had begun utilizing paper-based representations of fractions. However, the teacher referenced the benefit of being able to quickly pull up the website and use the virtual manipulatives to facilitate a discussion or address points of confusion.

Recommendations for Open-Ended Learning Environments and Virtual Manipulatives

There are multiple benefits to using Open-Ended Learning Environments (OELEs) and virtual manipulatives in mathematics classrooms. Of note, the benefit of mathematical reasoning and higher-level thinking skills was embedded in both lesson seeds (i.e., vignettes). While the teacher could have given the students a simpler one-step task or activity to do with technology, these classroom experiences involved mathematical tasks that required multiple steps and reasoning about fractions. In both of these contexts, the technology served as a tool that supported students' thinking and reasoning and did not do the thinking and problem solving for the students. While multiple mathematics-learning technologies have been designed to develop students' basic skills and computation work, research has cited that technology has a maximum benefit when students use it to generate and use mathematical pictures and representations related to the concepts that they are exploring (Howerton & Polly, 2023; McCulloch & Lovett, 2023; Zbiek et al., 2007).

Another noteworthy part of these lesson seeds (vignettes) is the teacher actions included with the use of the virtual fraction bars. The teacher facilitated learning by launching a task and ensuring that students understood what the task was asking them to do. Additionally, the teacher observed students and asked a lot of questions to observe and notice how students were exploring the task and to help clarify their thinking. As is the case with lesson seed (vignette) #2 with the comparison of fractions, the teacher's questions helped students to make more sense of what they did on paper as they orally talked about their strategies and mathematical ideas.

For teachers who may be anxious or unsure about having their students explore mathematics concepts using virtual manipulatives, this idea can be started with small groups of students instead of first working with whole groups (Howerton & Polly, 2023; Polly & Castillo, 2024; Polly & Howerton, 2023; Urbina & Polly, 2017). This approach allows teachers to be in close proximity with students as they work to help them troubleshoot any technology complexities and also to ask questions to support their learning.

Lastly, it must be reiterated that students did the "heavy lifting" and the majority of the work in these vignettes. Students used the technology, reasoned mathematically, and communicated their thinking. As learners of mathematics concepts, students must be given the opportunity, agency, and support to use technology in meaningful ways to help them deepen their understanding of mathematics concepts (Polly & Martin, 2024b).

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Exploring Fraction Concepts *(continued)*

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Drew Polly teachers courses at the University of North Carolina Charlotte in the elementary education program and supports mathematics teaching around North Carolina with professional learning experiences and curricula resources.